Who Plays First?

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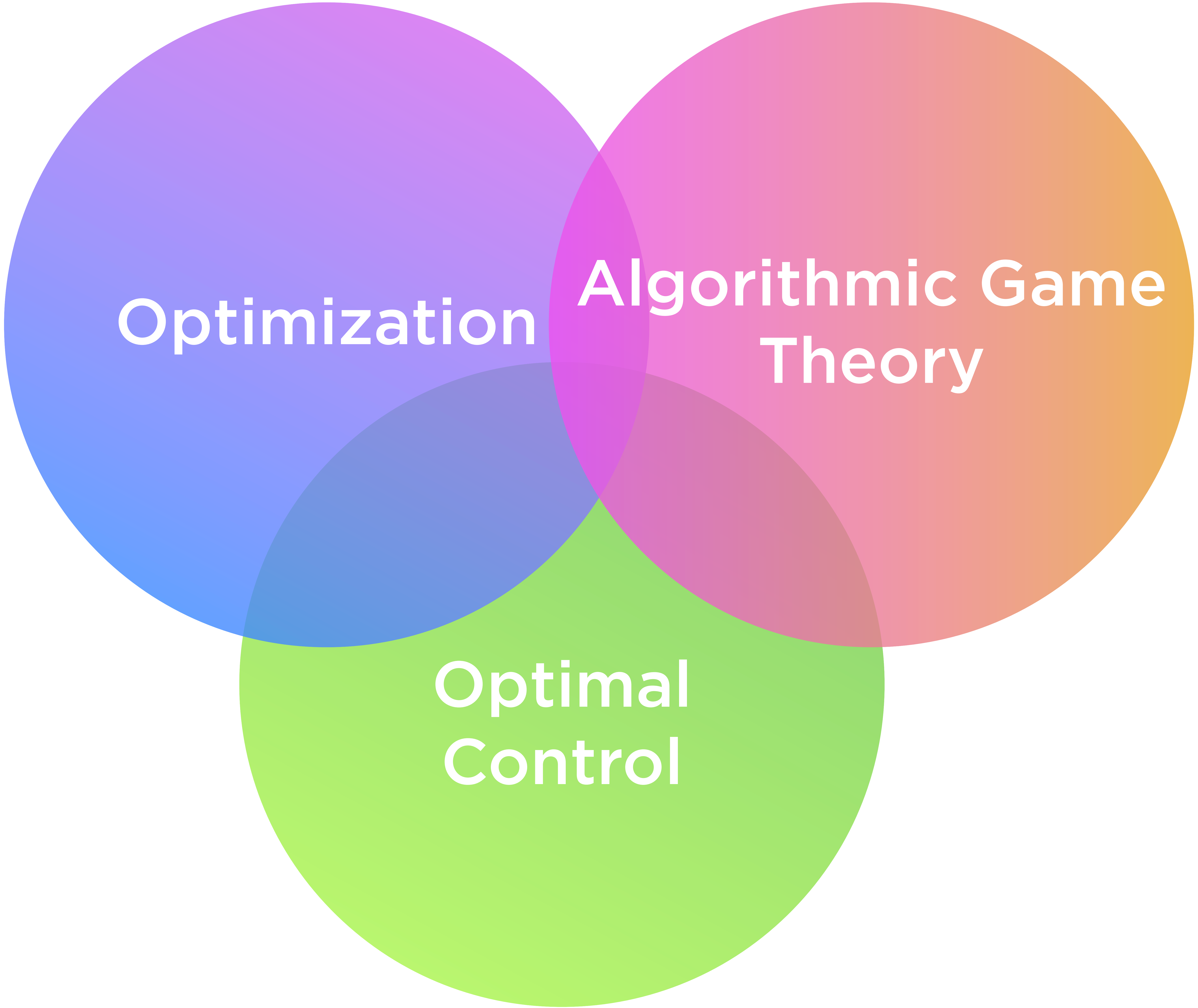

Optimization
Algorithmic Game Theory
Optimal Control

Regulating Airspace

is a complex FAA business…

Even more complex for UAVs...

Extremely complex when a myriad of ***autonomous*** UAVs interact

Preference and safety tradeoff?

Users

Want to **optimize their operations in real time**

- Avoid idle times,
- Match operations to their schedule,
- Plan trajectories
- Many players!

Regulator

Wants to (primarily) **ensure safety**

- **Collision-free** planning,
- Ensure fairness,
- Then, optimize efficiency.

Conflicting objectives

Wants to (primarily) **ensure safety**

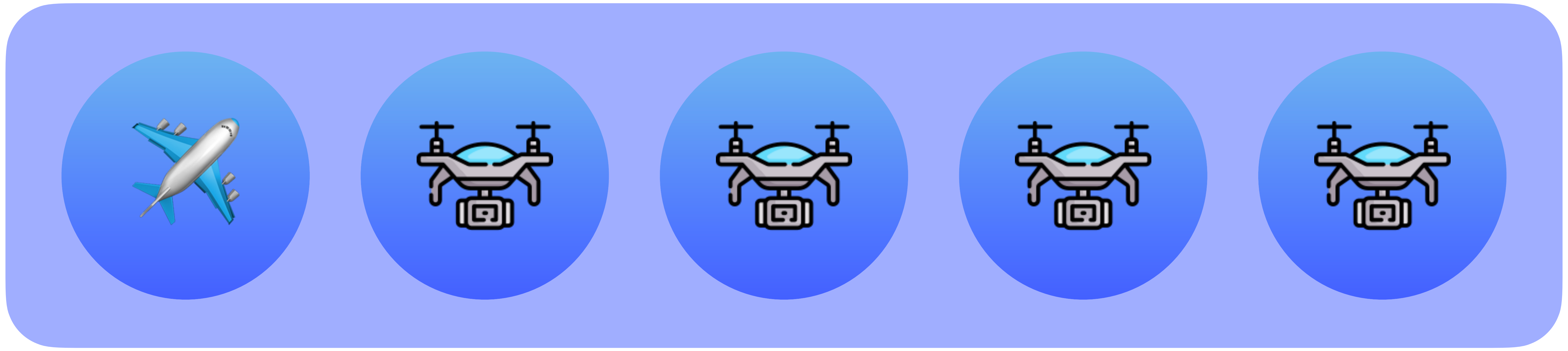
Set the **incentives**

Optimize their operations according to the incentives

The most natural incentive

How to determine the **order of play** (i.e., the order of trajectory planning) such that the **regulator optimizes its objectives**?

Who Plays First?



Problem abstraction

Enforces the order by setting $z_p = 1$ for some permutation $p \in \mathcal{P}$

Optimizes a cost function $f(x^1, \ldots, x^n) = f(x)$

Variables of all players but i

Play a strategy $x \in \mathcal{X}(z, x^{-i})$ factoring the order and the other players' strategies

Optimizes a cost function $g^i(x^i; x^{-i}, z)$

$$
\begin{aligned}
\min_{x,z,\hat{x}} \quad & f(x) \\
\text{subject to} \quad & (x_p^1, \ldots, x_p^n) \in \mathcal{S}(p), \qquad \forall p \in \mathcal{P} \\
& x = \sum_{p \in \mathcal{P}} z_p \hat{x}_p, \\
& z \in \{0,1\}^{|\mathcal{P}|}, \quad \sum_{p \in \mathcal{P}} z_p = 1
\end{aligned}
$$

"Blackbox"

Given a permutation

$$\text{subject to} \quad (x_p^1, \ldots, x_p^n) \in \mathcal{S}(p), \qquad \forall p \in \mathcal{P}$$

Explicit Blackbox

$\mathcal{S}(p)$ **is a linear system**

Given z, player i solves a **convex-quadratic equality-constrained Stackelberg game**

$$\min_{x^i} \quad g^i(x^i; x^{-i}, z) := \frac{1}{2}(x^i)^\top Q^i x^i + (c^i)^\top x^i + (x^{-i})^\top C^i x^i$$

Sequential "Stackelberg" (or n-level) game

$$x_{after}^{-i} \in OPT(x^{-i}, x^i)$$

"Followers" optimality

$$x^i \in \mathbb{R}^m$$

Implicit Blackbox

A blackbox can provide us the solution for the players

Given a permutation

$$\text{subject to} \quad (x_p^1, \ldots, x_p^n) \in \mathcal{S}(p), \qquad \forall p \in \mathcal{P}$$

Sequential Convex QP Game

$\mathcal{S}(p)$ **is a linear system**

Given z, player i solves a **convex-quadratic equality-constrained Stackelberg game**:

$$\begin{aligned} \min_{x^i} \quad & g^i(x^i; x^{-i}, z) := \frac{1}{2}(x^i)^\top Q^i x^i + (c^i)^\top x^i + (x^{-i})^\top C^i x^i \\ \text{subject to} \quad & A^i x^i + D^i x^{-i} - b^i = 0, \\ & x_{after}^{-i} \in OPT(x^{-i}, x^i) \\ & x^i \in \mathbb{R}^m \end{aligned}$$

"Followers" optimality

Blackbox

A blackbox can provide us the **solution for a given p**

STP converges to a Stackelberg equilibrium

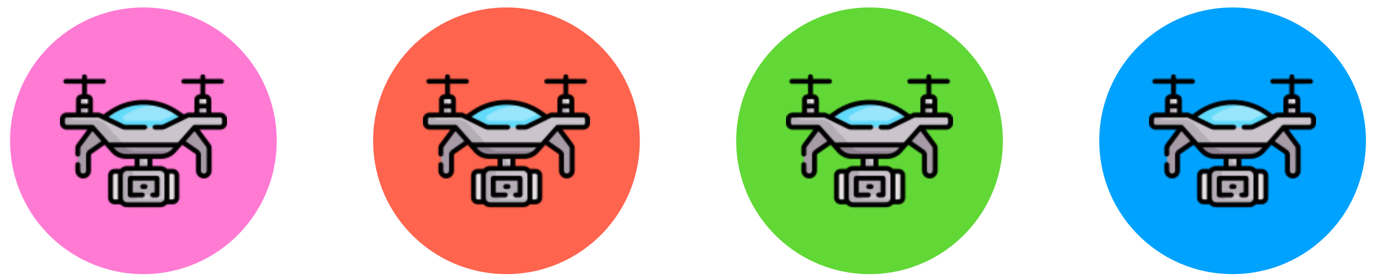
Sequential trajectory planning (STP), a popular multi-agent control approach, **produces a local Stackelberg equilibrium** in a single pass

Deterministic

Linear wrt n

Supports constraints

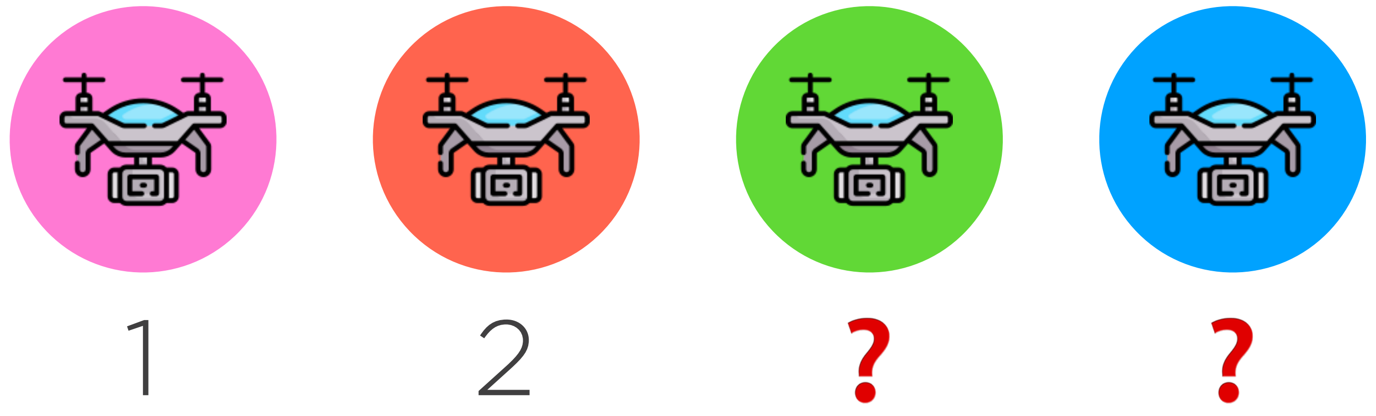
Two fundamental operations



1 **Partial Permutation**

A **partial assignment** $\tilde{p}$ of the order of play

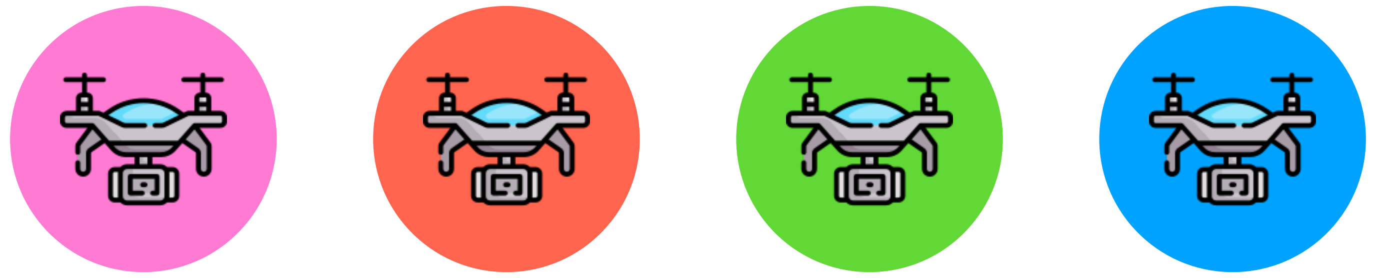
i.e., we know the first $k < (n-1)$ players playing



2 **Pricing $\tilde{p}$**

Assign an optimistic f-bound on $\tilde{p}$ **monotonically-increasing** w.r.t. any children

Pricing, in depth



2 **Pricing $\tilde{p}$**

Assign an optimistic f-bound on $\tilde{p}$ **monotonically-increasing** w.r.t. any children

Assuming the UAV i plays as the l-th player, it solves

$$
\begin{aligned}
\text{minimize} \quad & g^i(x^i; x^{-i}) := \textstyle\sum_{t=0}^{T} \ell^i(x_t^i, u_t^i) \\
\text{subject to} \quad & x_{t+1}^i = f^i(x_t^i, u_t^i) \\
& x_0^i = x_{init}^i \\
& x_t^i \in \mathcal{X}^i, \quad u_t^i \in U^i \\
& x_t^i \notin \mathcal{O}^i
\end{aligned}
$$

Trajectory Dynamics

Set of controls

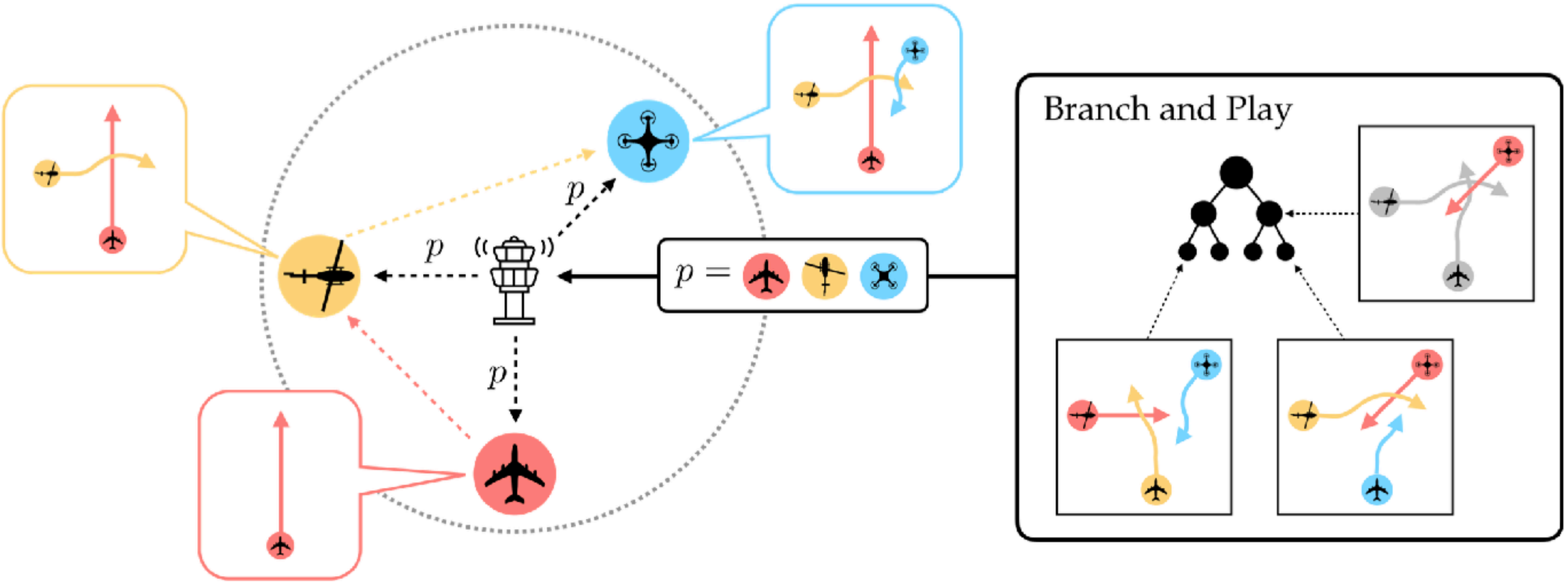
Obstacles and no-fly zones

The Branch-and-Play

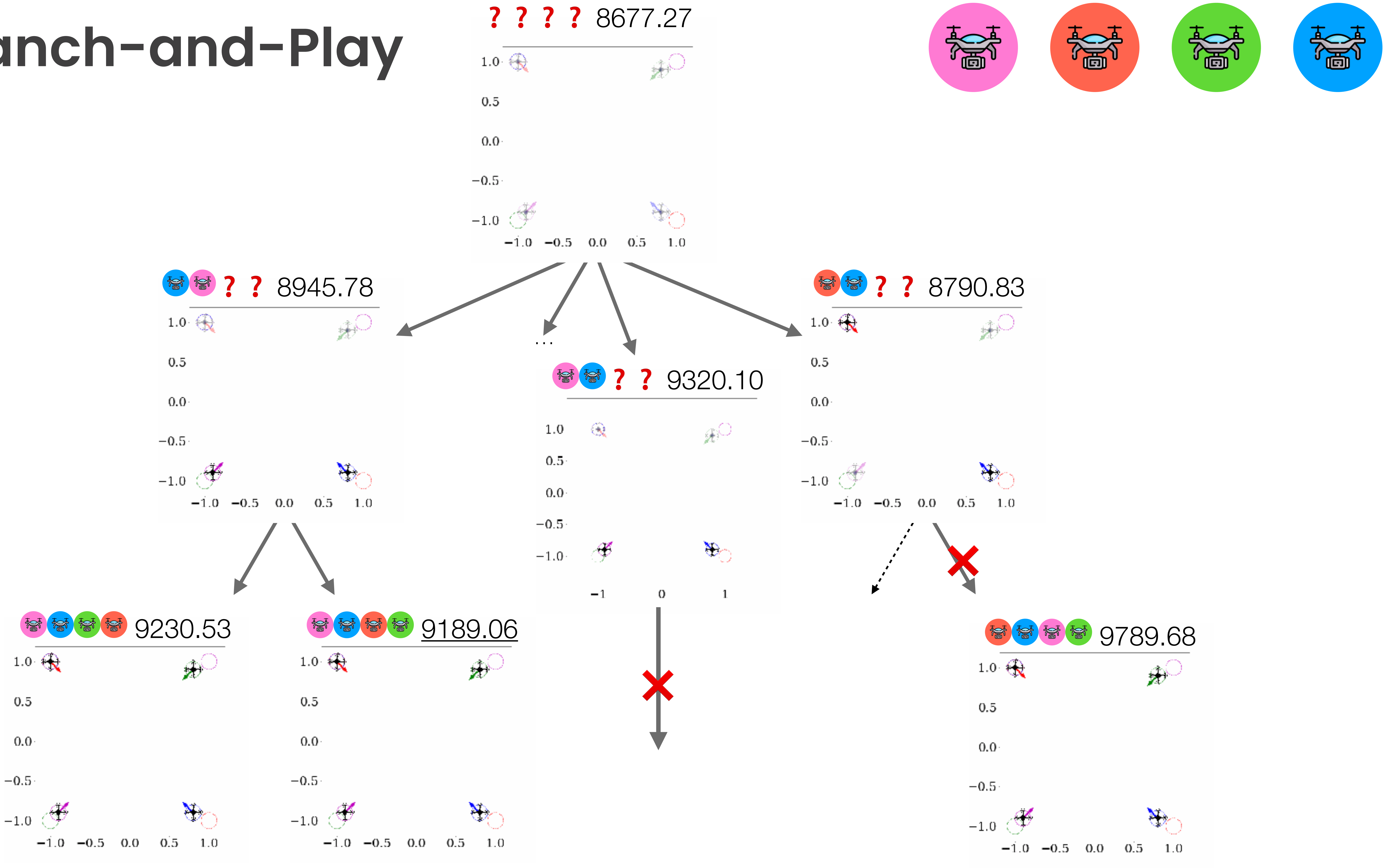
Branch-and-Play

Implicitly enumerate the space of permutations by:

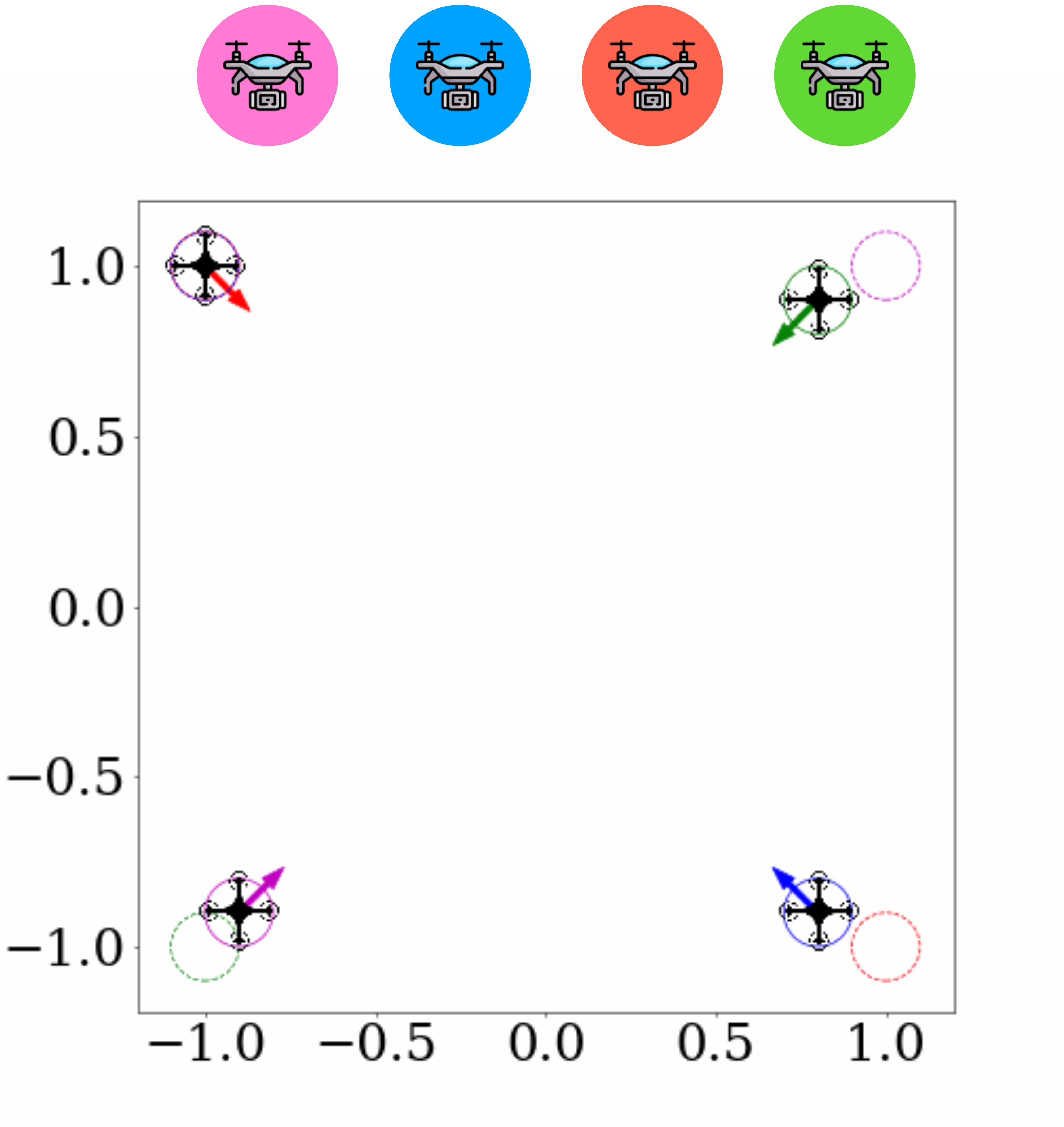
- **Pricing** of partial permutations
- **Pruning and exploration** strategies



The Branch-and-Play



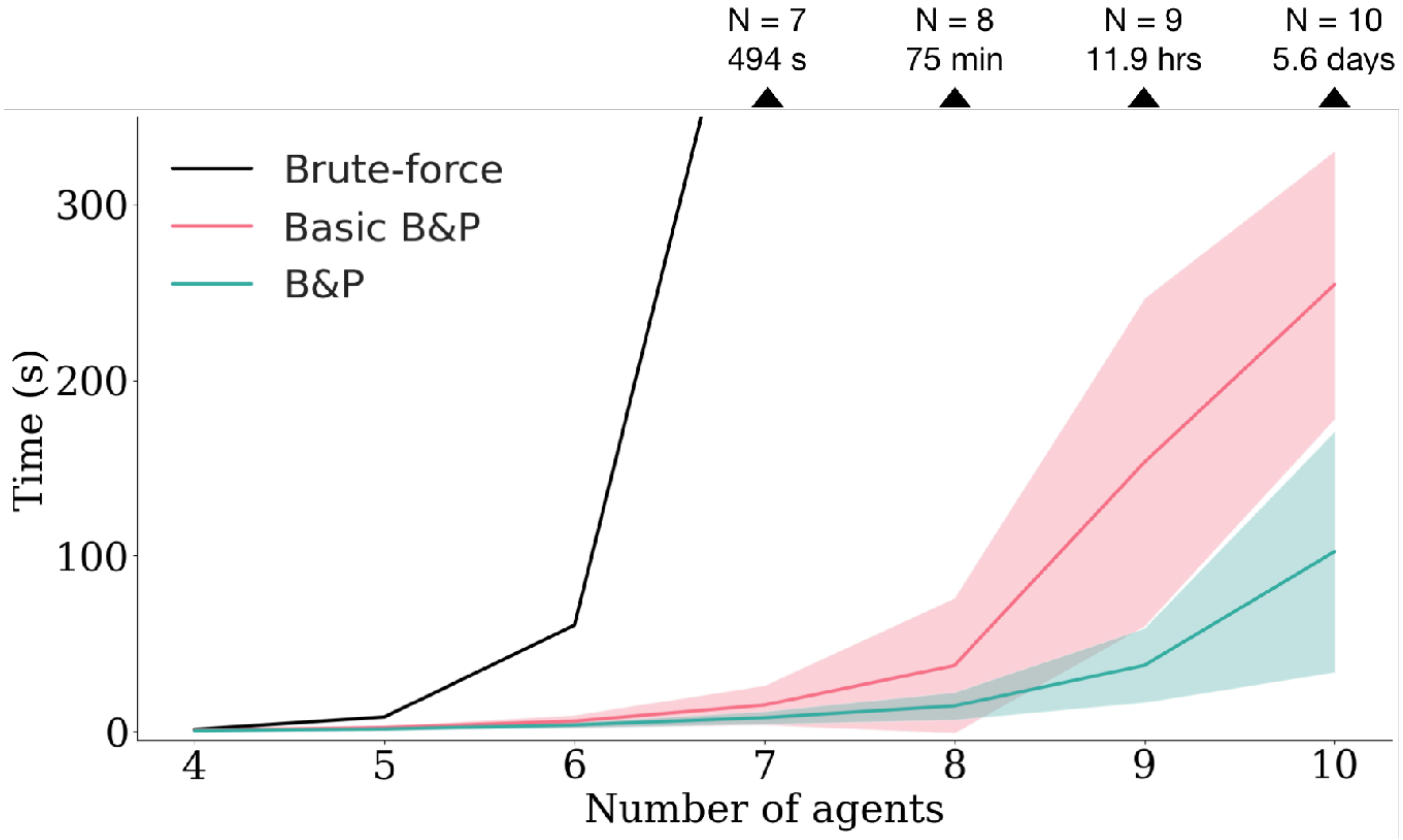
The Branch-and-Play



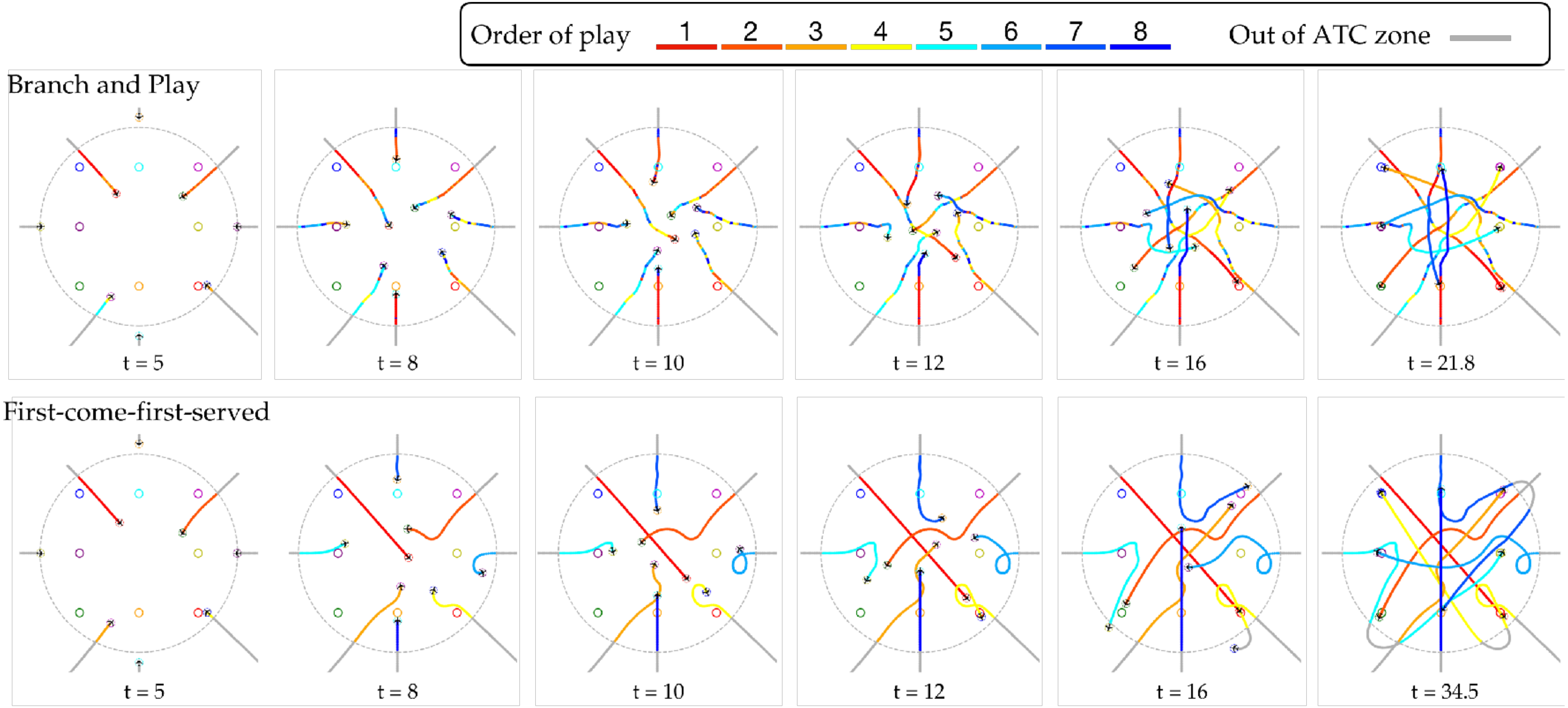
f-bound of <u>9189.06</u>

Experiments

BnP performs time-wise consistently in ATC



Visualizing ATC trajectories



Better flight times with ATC

Metric	N	FCFS	Randomized	Nash ILQ	B&P (ours)
Cost		1.24 ± 0.29	1.19 ± 0.23	3.61 ± 0.94	$\mathbf{1.04 \pm 0.24}$
Group (s)	4	25.76 ± 5.0	24.98 ± 4.06	44.50 ± 8.84	$\mathbf{20.41 \pm 2.72}$
T/O rate		1%	**0%**	36%	**0%**
Cost		1.73 ± 0.49	1.70 ± 0.43	5.40 ± 1.36	$\mathbf{1.41 \pm 0.3}$
Group (s)	5	29.27 ± 8.29	28.84 ± 6.74	48.34 ± 11.4	$\mathbf{20.5 \pm 6.04}$
T/O rate		6%	2%	64%	**1%**
Cost		2.43 ± 0.82	2.55 ± 1.09	8.73 ± 2.13	$\mathbf{2.0 \pm 0.51}$
Group (s)	6	31.83 ± 9.34	33.62 ± 12.05	50.9 ± 11.92	$\mathbf{22.18 \pm 7.74}$
T/O rate		9%	20%	96%	**2%**

Branch-and-Play **outperforms 3 baselines** in all three metrics

Swarm formation

Autonomous vehicles

Modular Algorithm — Adapts to a variety of applications. Enables **customizations**

Actionable Insights — Provides **interpretable interventions**

Solid Benchmark Results — Guarantees **tangible improvements** over the SOTA

www.dragot.to

Who Plays First? Optimizing the Order of Play in Stackelberg Games with Many Robots

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DataX — Accelerating Scientific Discovery at Princeton

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