

Mathematical Programming Games

Gabriele Dragotto
PDC 2022 - November 15, 2022



Operations Research &
Financial Engineering



PRINCETON
UNIVERSITY

Without Math!

~~Mathematical Programming Games~~

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Operations Research &
Financial Engineering



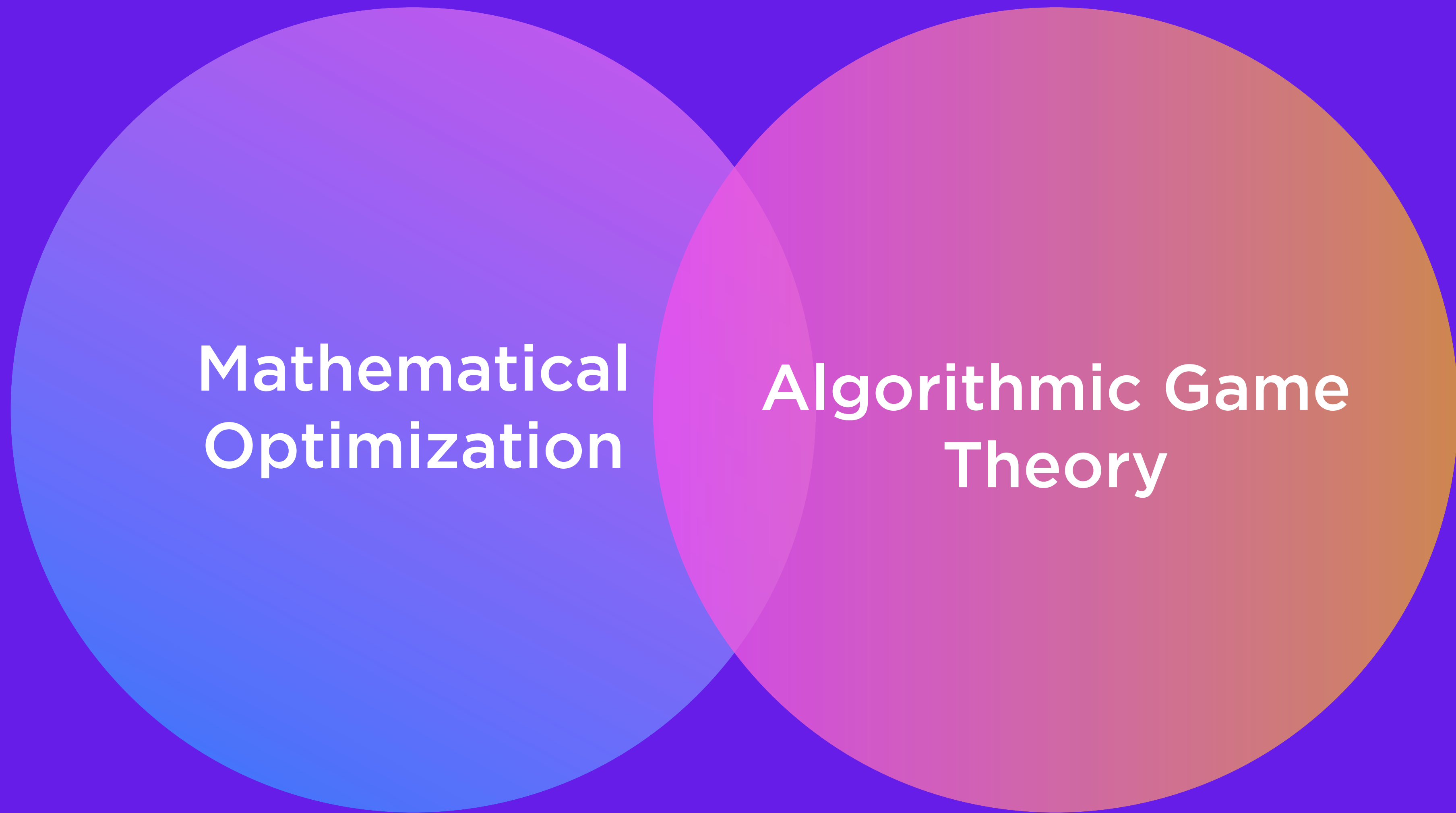
PRINCETON
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A Venn diagram consisting of two overlapping circles on a solid purple background. The left circle is a light blue color and contains the text 'Mathematical Optimization'. The right circle is a light orange color and contains the text 'Algorithmic Game Theory'. The overlapping area in the center is a darker purple color.

**Mathematical
Optimization**

**Algorithmic Game
Theory**



**Mathematical
Optimization**

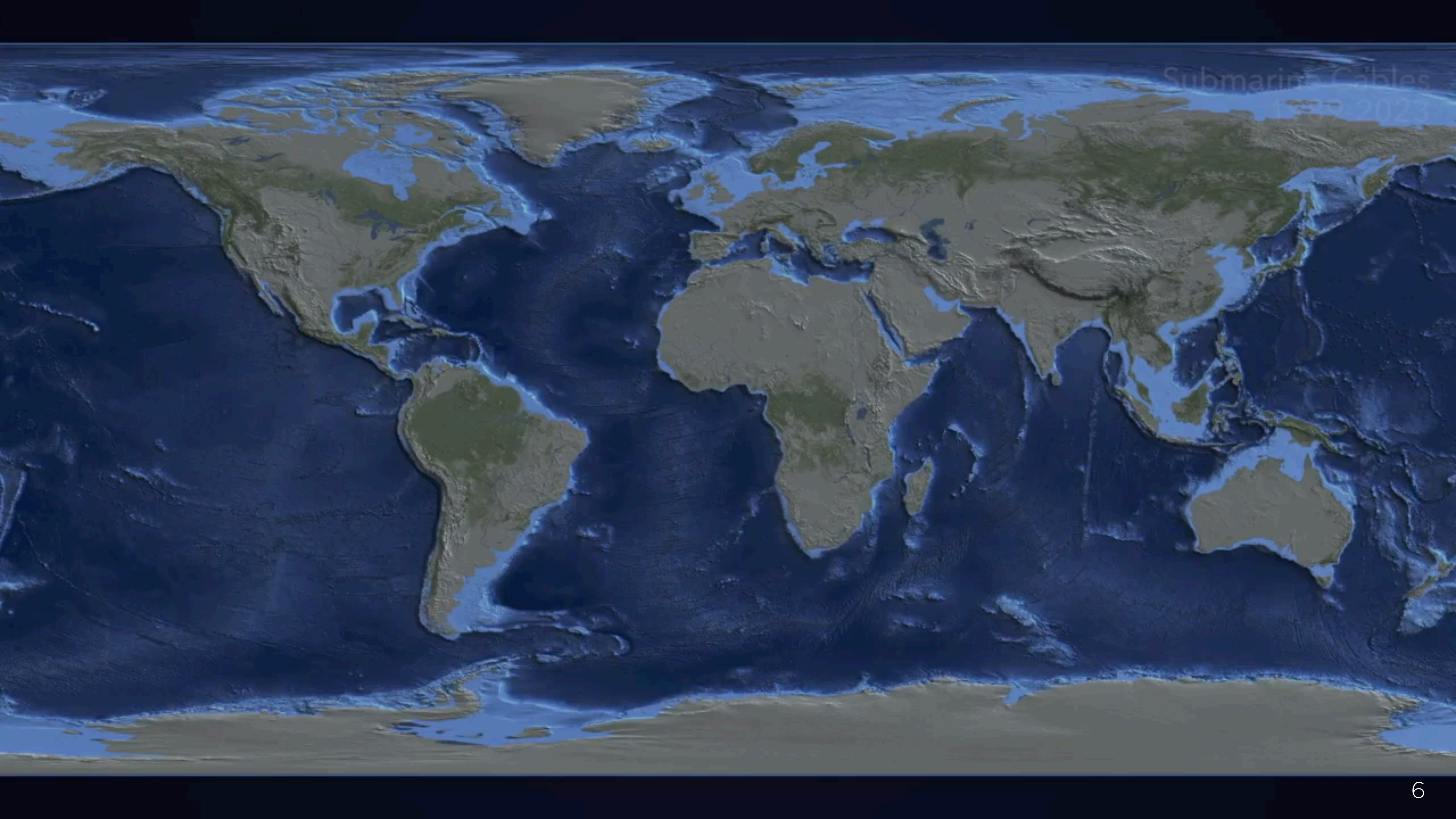
**Algorithmic Game
Theory**

Decision-making

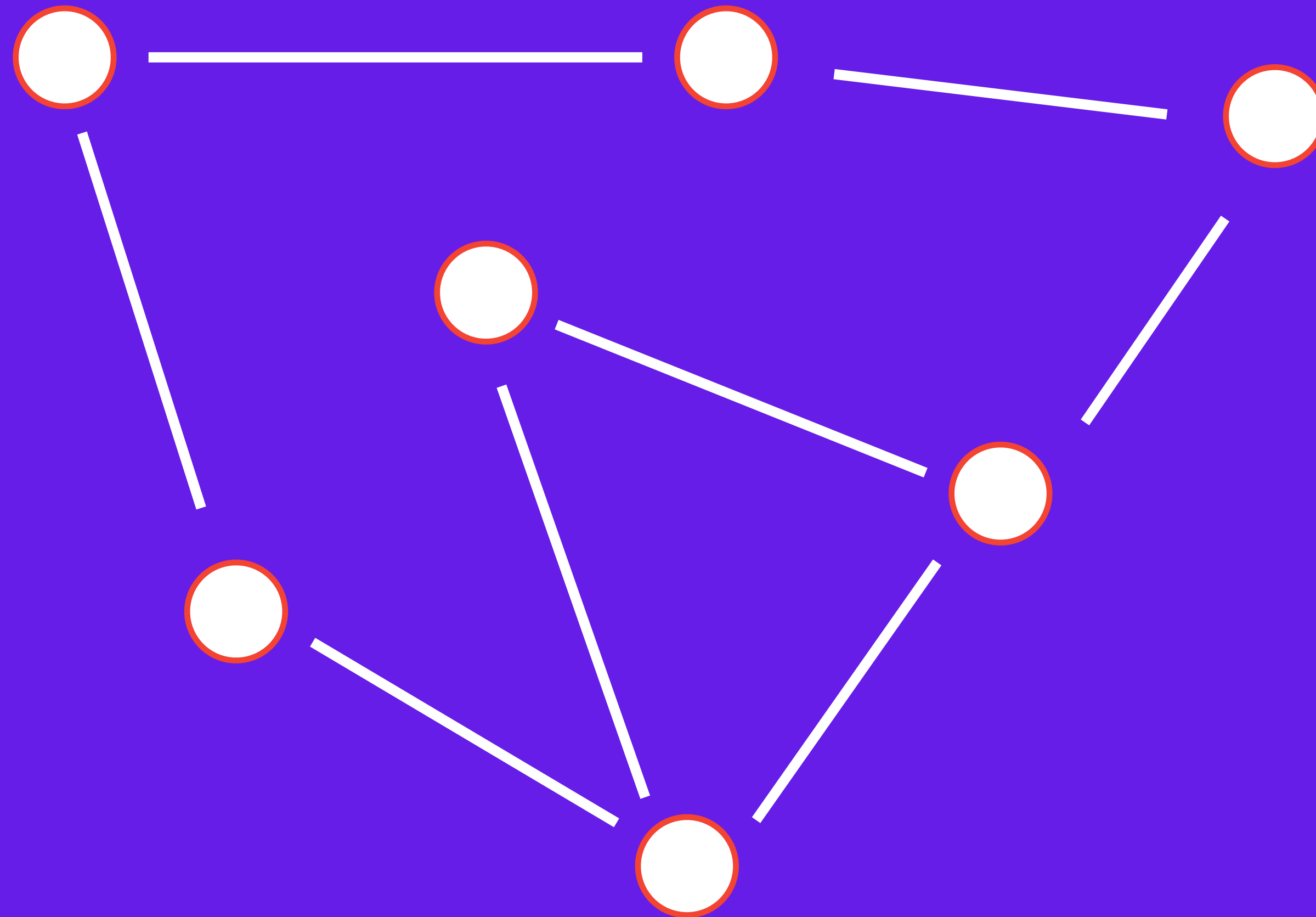
Strategic Decision-making

The background of the slide is a photograph of the Oculus structure at the World Trade Center, viewed from a low angle looking up. The entire image is covered with a semi-transparent purple overlay. The text "Network Formation" is centered in white.

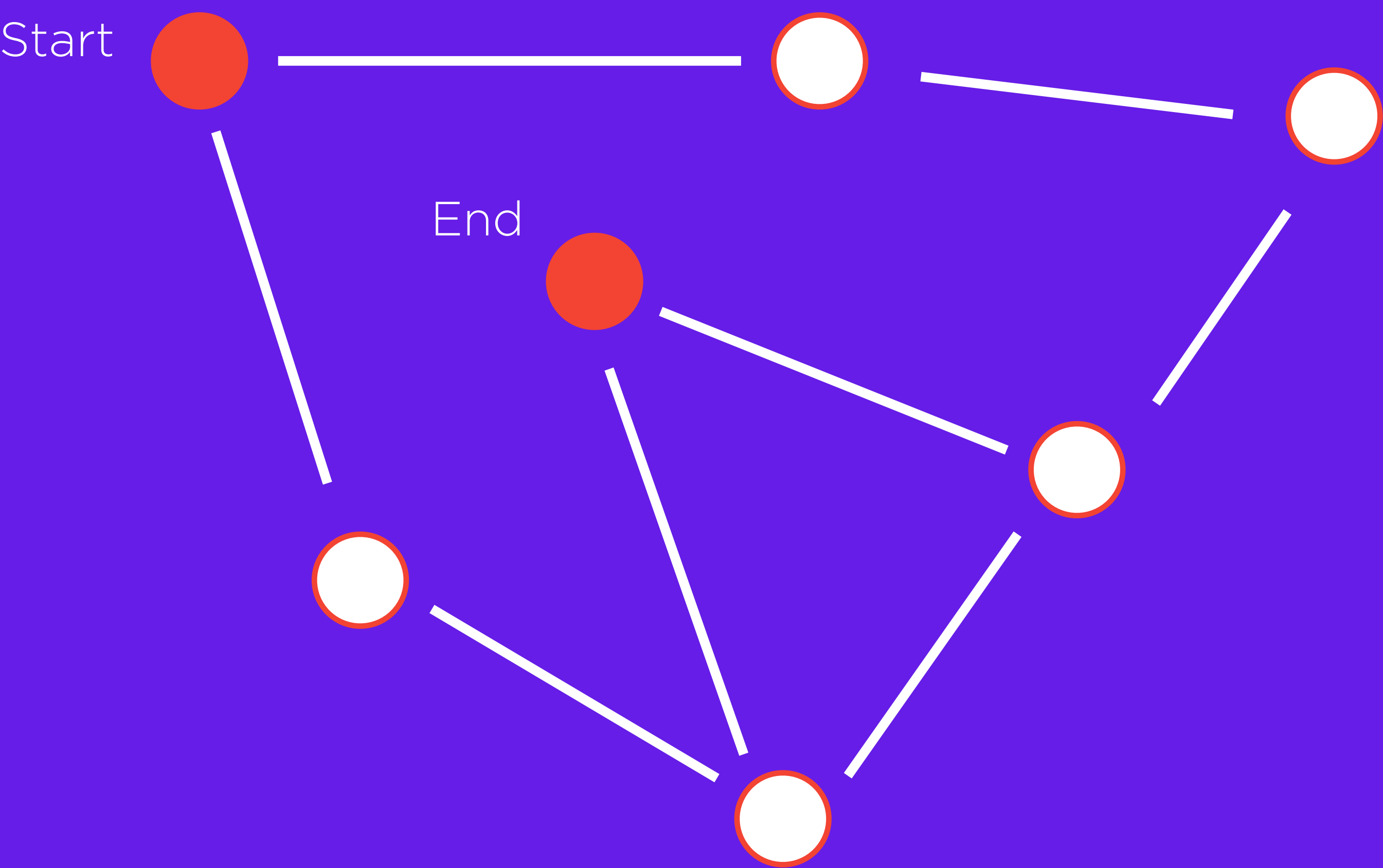
Network Formation



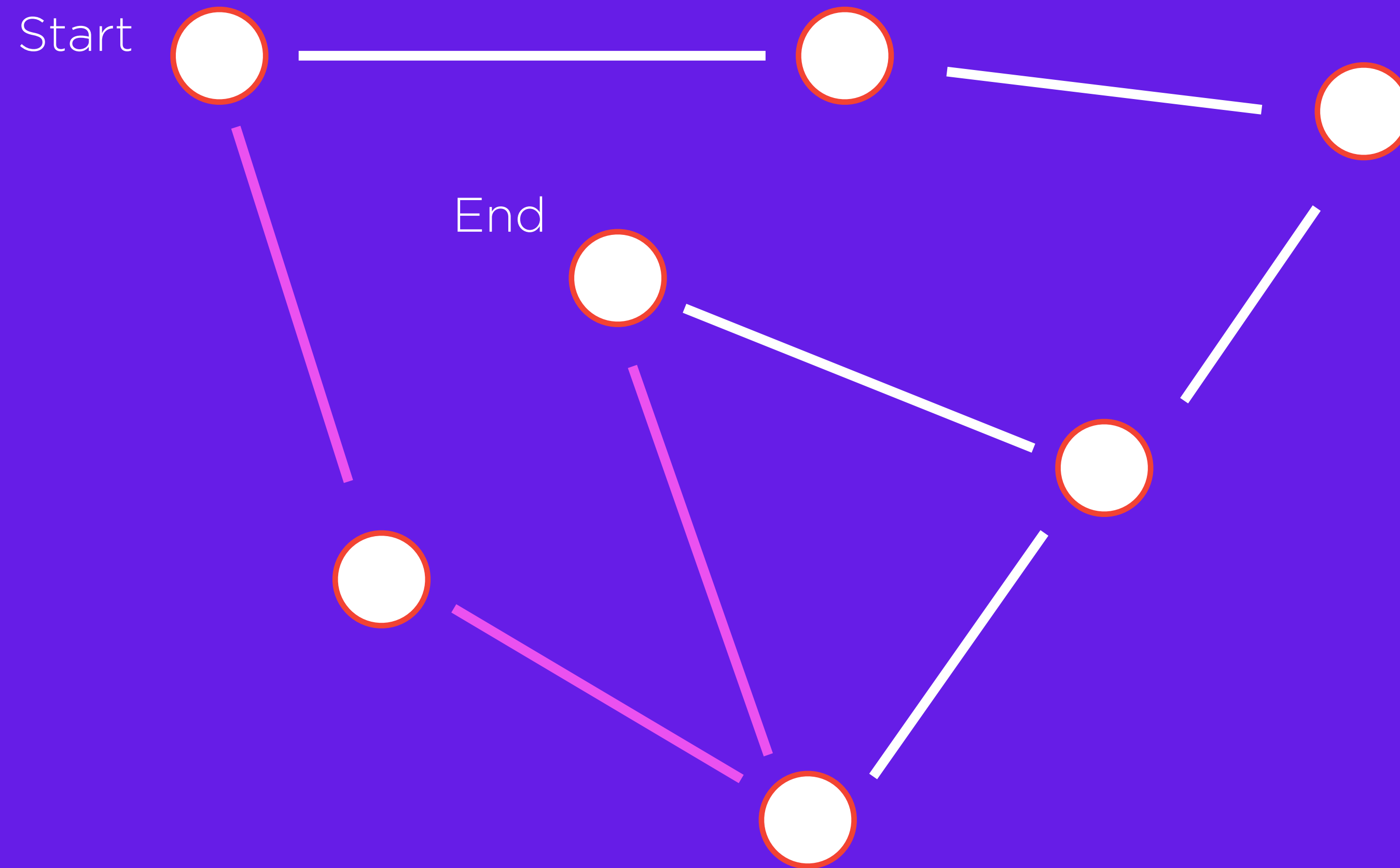
Network Formation Game



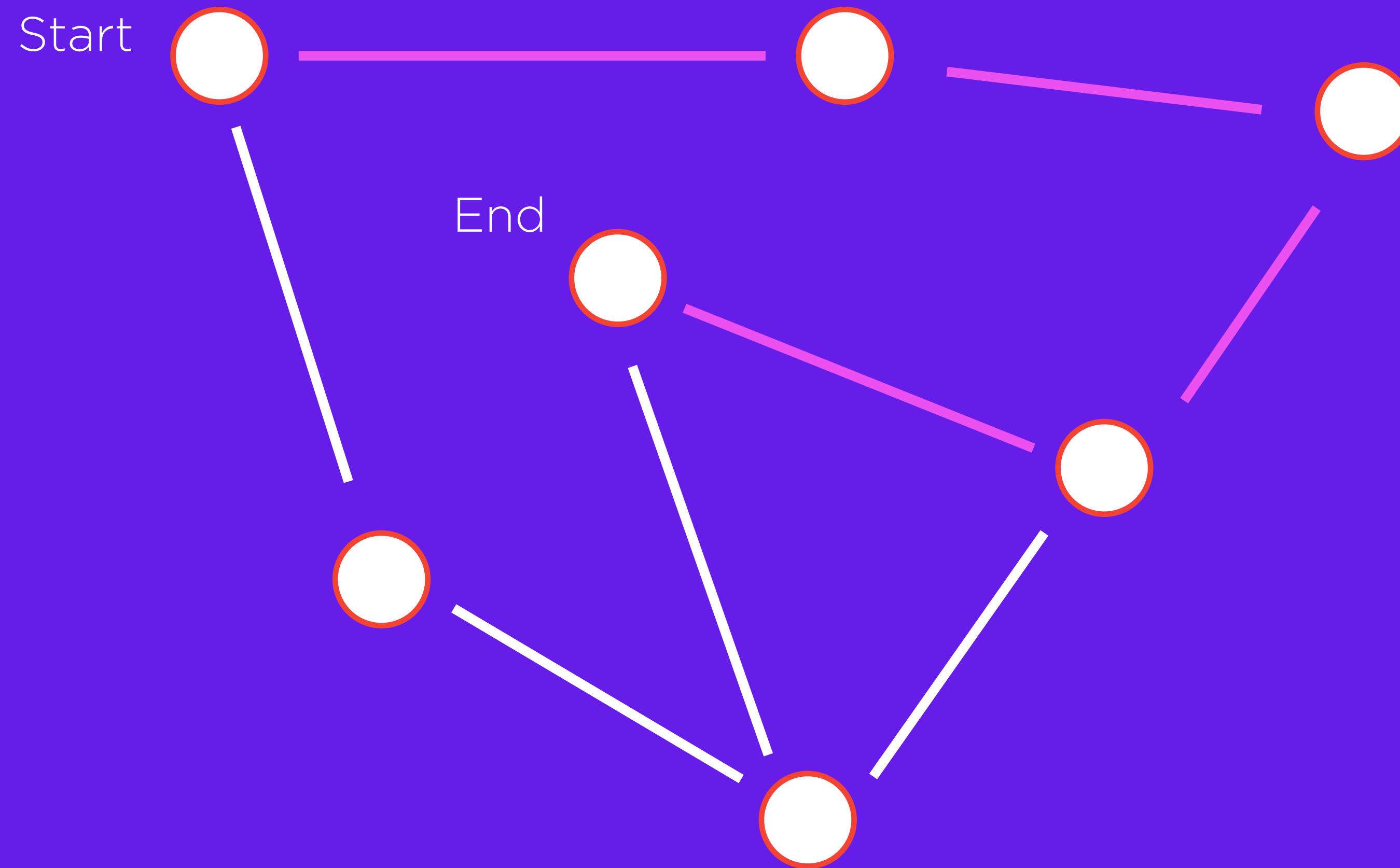
Network Formation Game



Network Formation Game

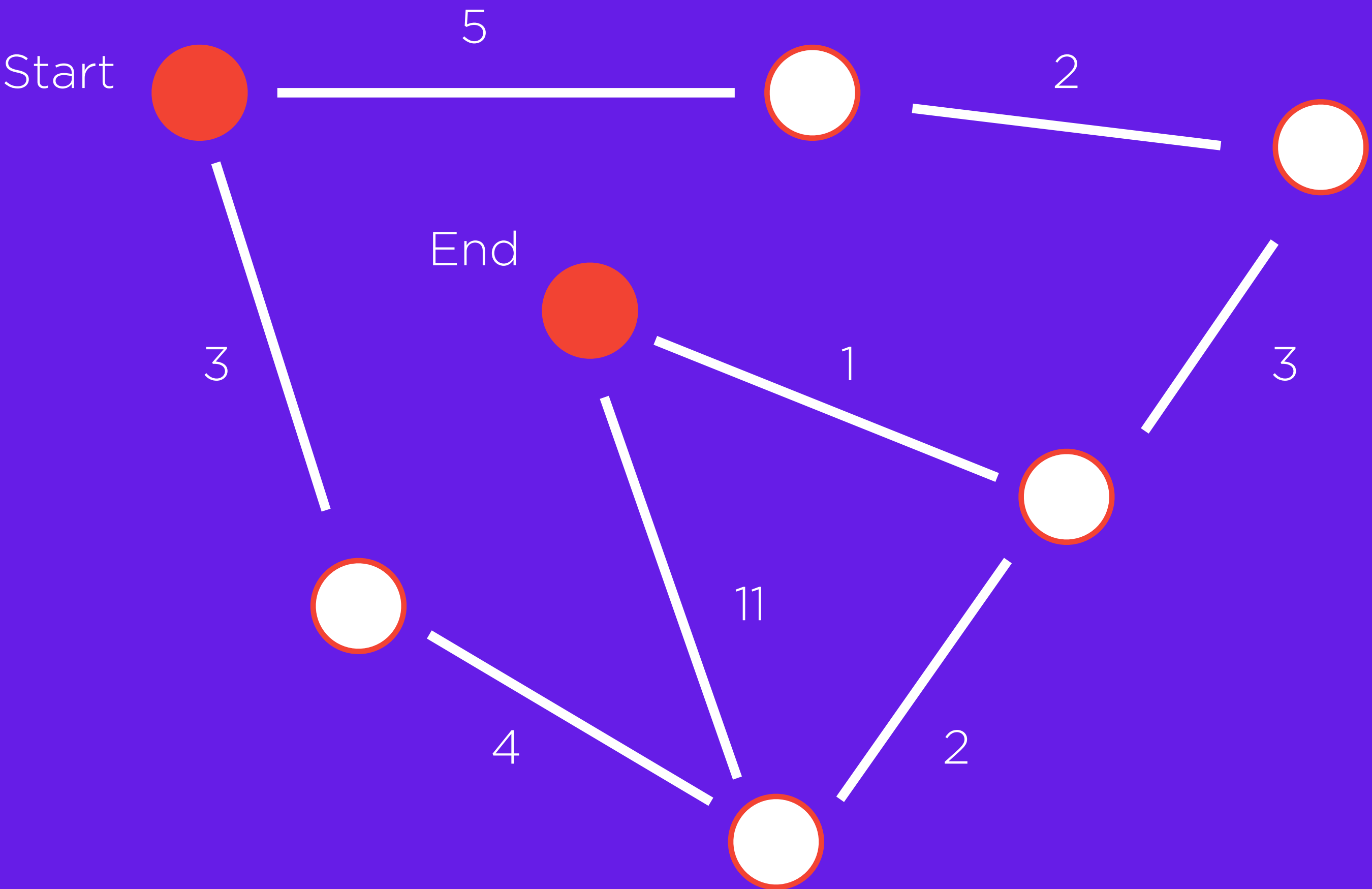


Network Formation Game

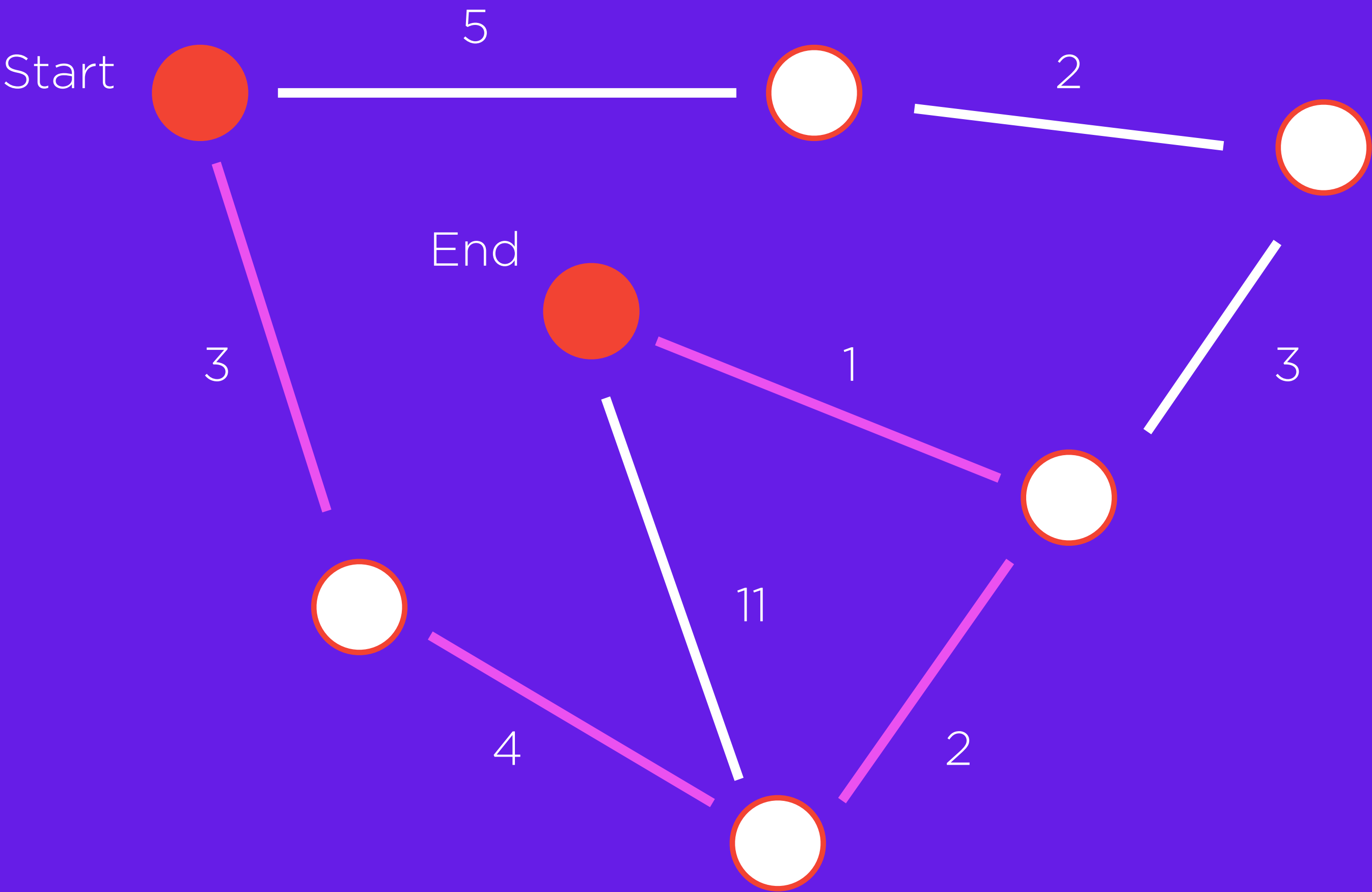


Costs

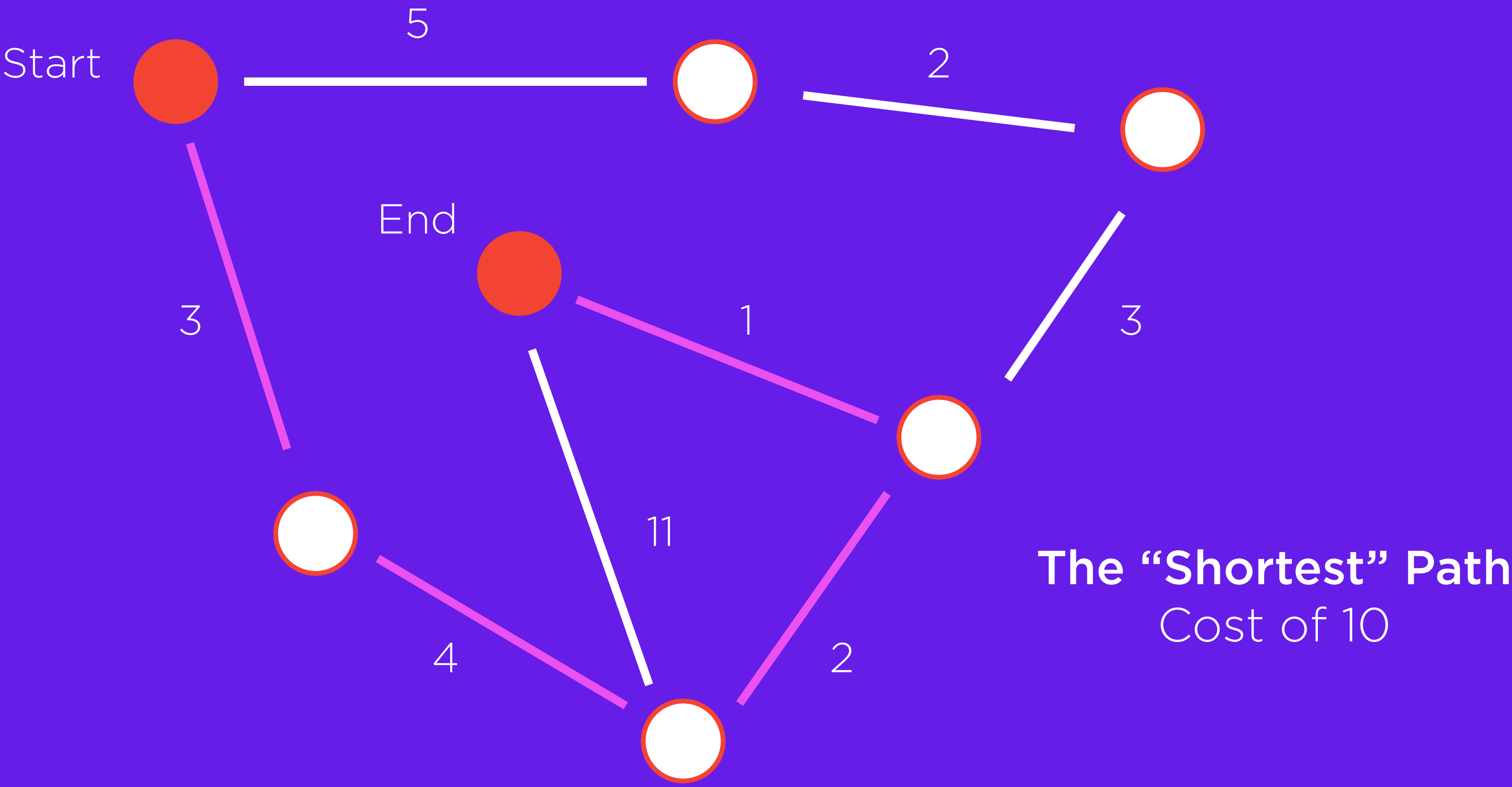
Network Formation Game



Network Formation Game

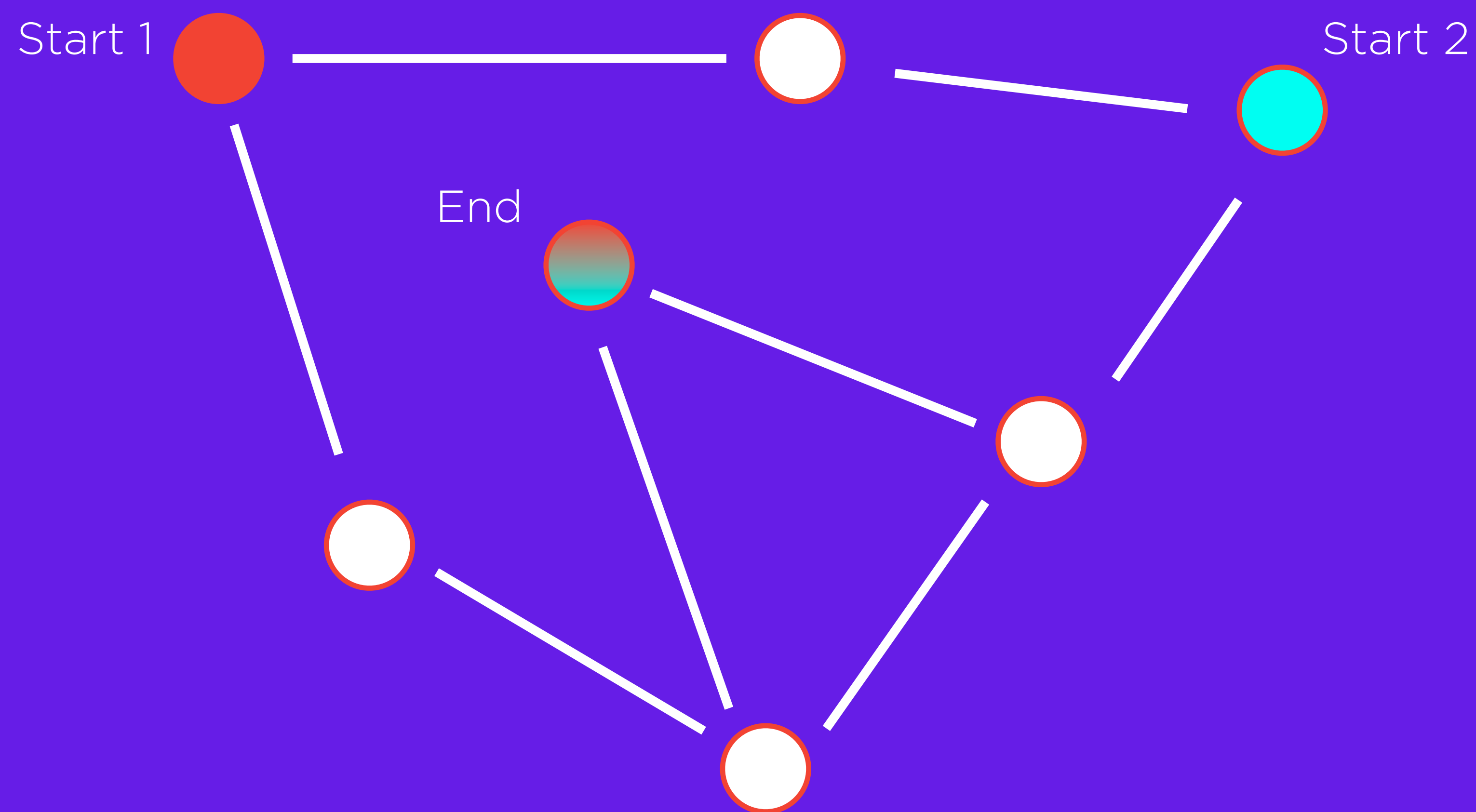


Network Formation Game

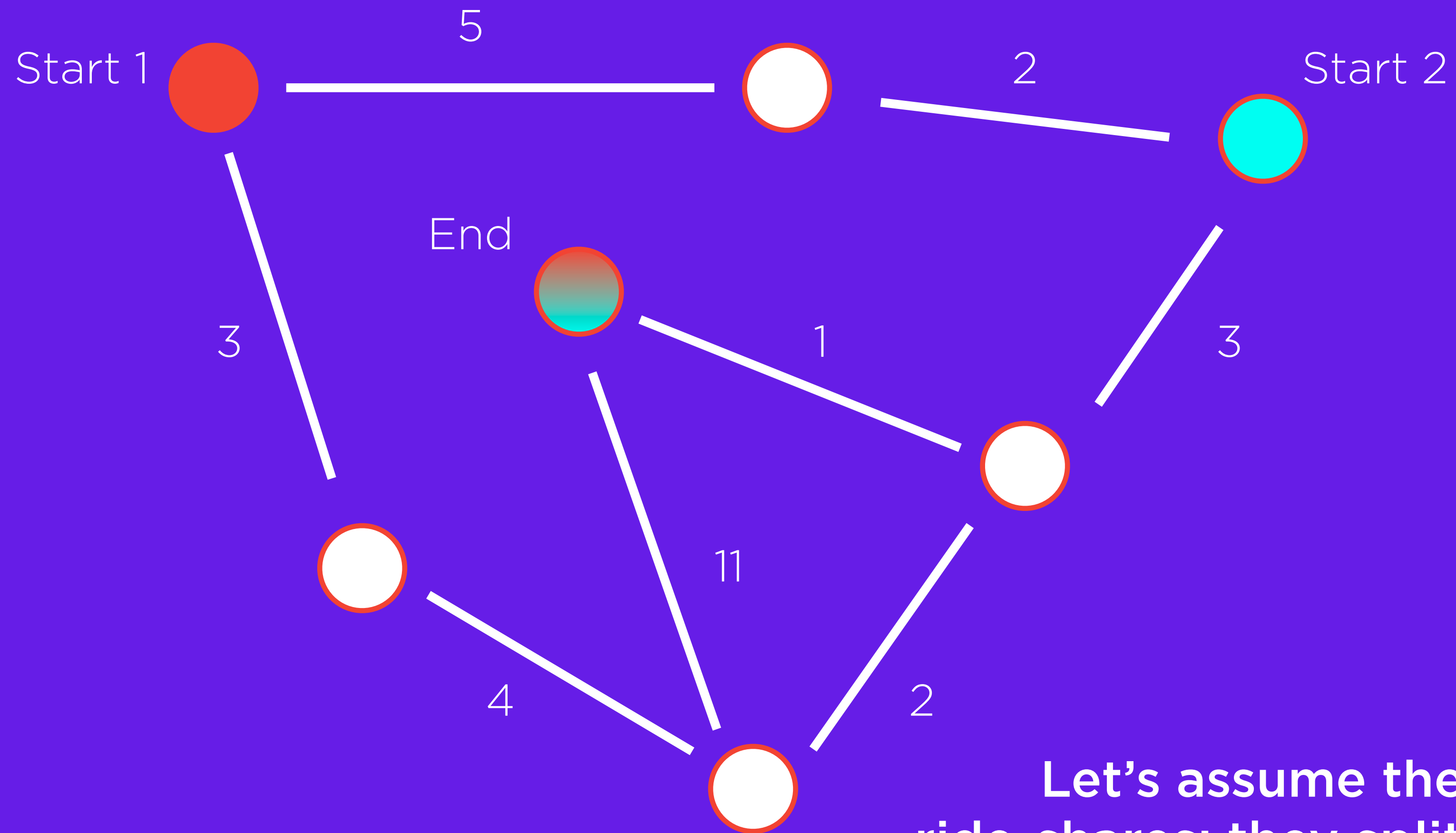


Interactions

Network Formation Game

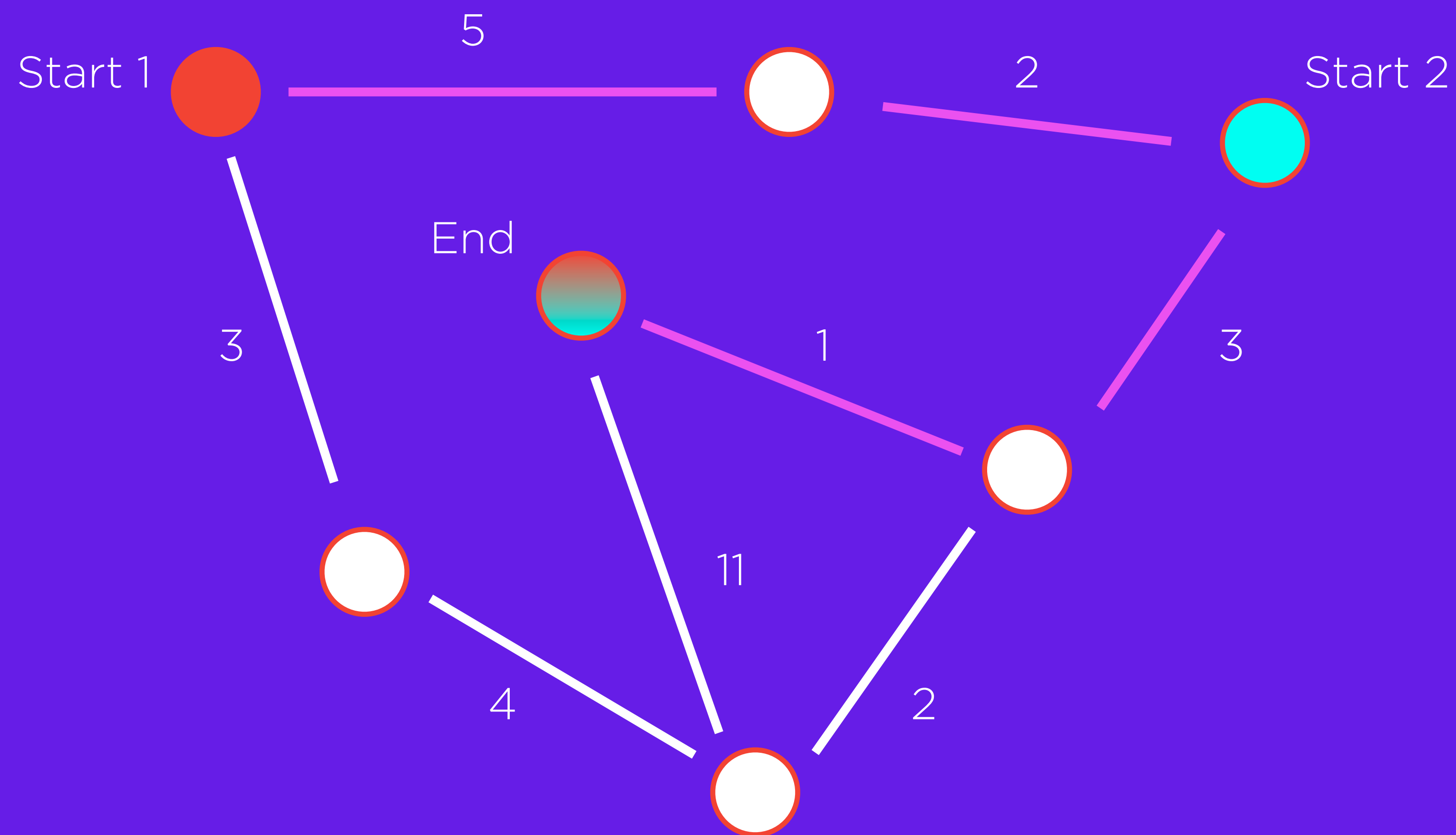


Network Formation Game

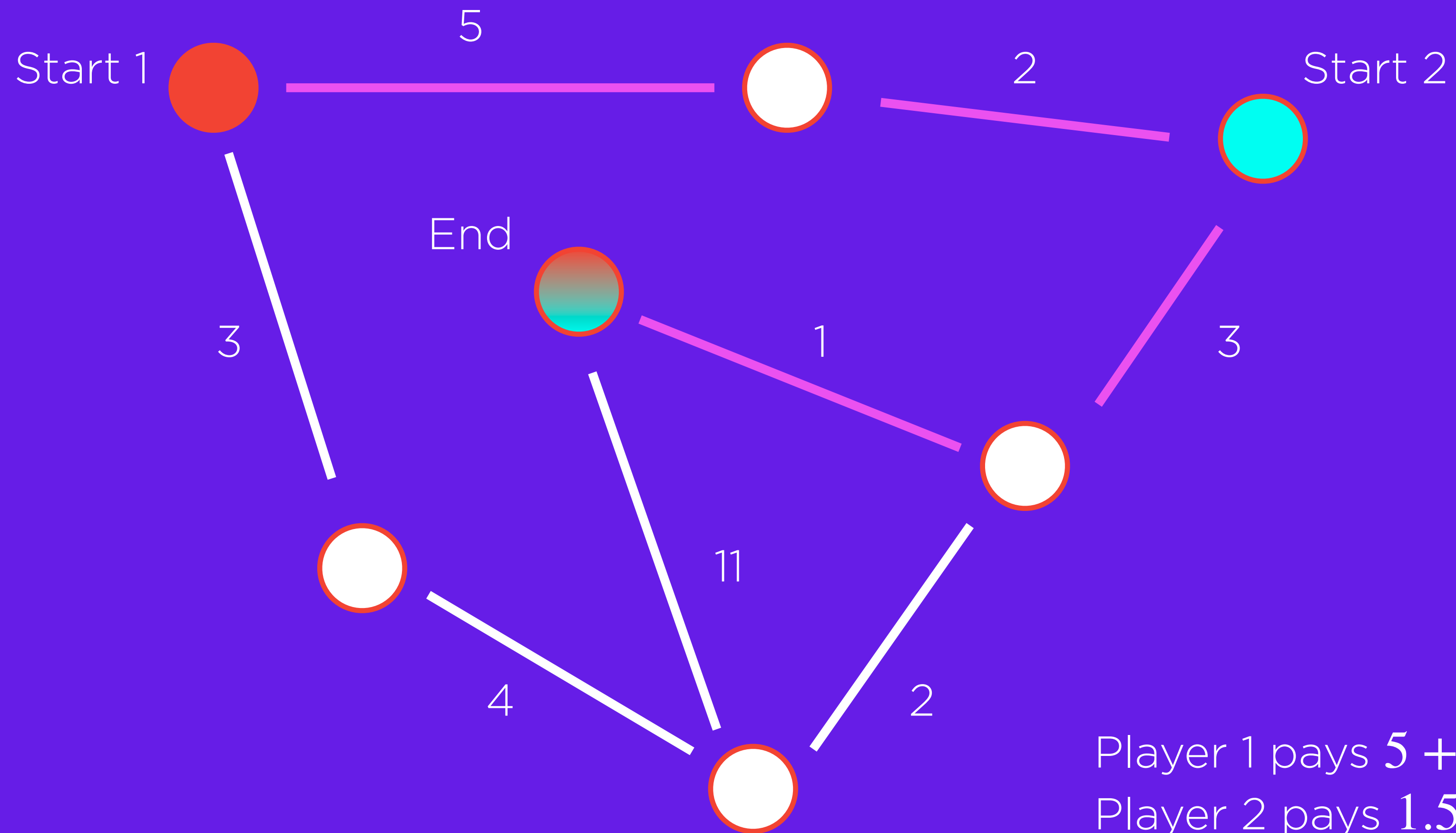


Let's assume the players are ride-shares: they split equally the cost

Network Formation Game



Network Formation Game





Vaccines

Coronavirus

EU threatens to block Covid vaccine exports amid AstraZeneca shortfall

Coronavirus

Macron calls for Covid vaccine exports from EU to be controlled

Coronavirus

EU could block millions of Covid vaccine doses from entering UK

How EU's floundering vaccine effort hit a fresh crisis with exports row

theguardian



Consider a *Drug*

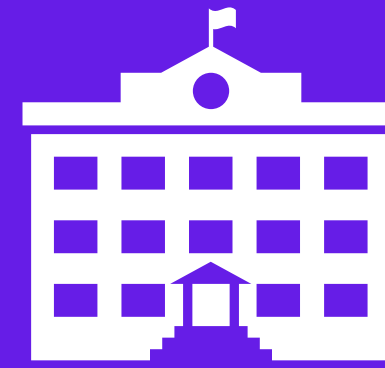


Pfizer

produces and sells its *Drug* in a market in order to profit

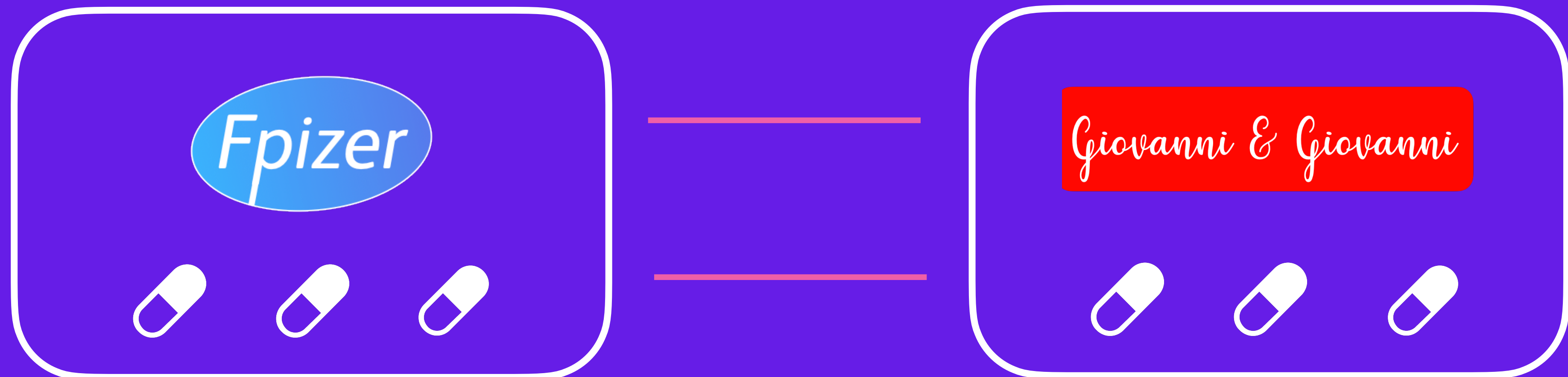


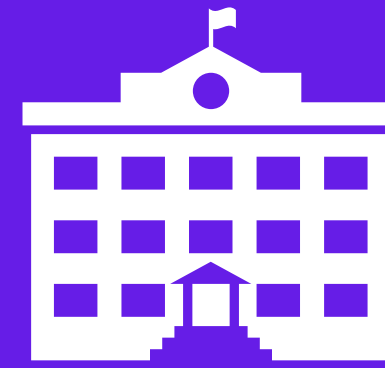
And competes with Giovanni & Giovanni
In a market of equivalent drugs



Canada taxes their drugs

And regulates exports/imports of the drug





Hierarchical
Regulation

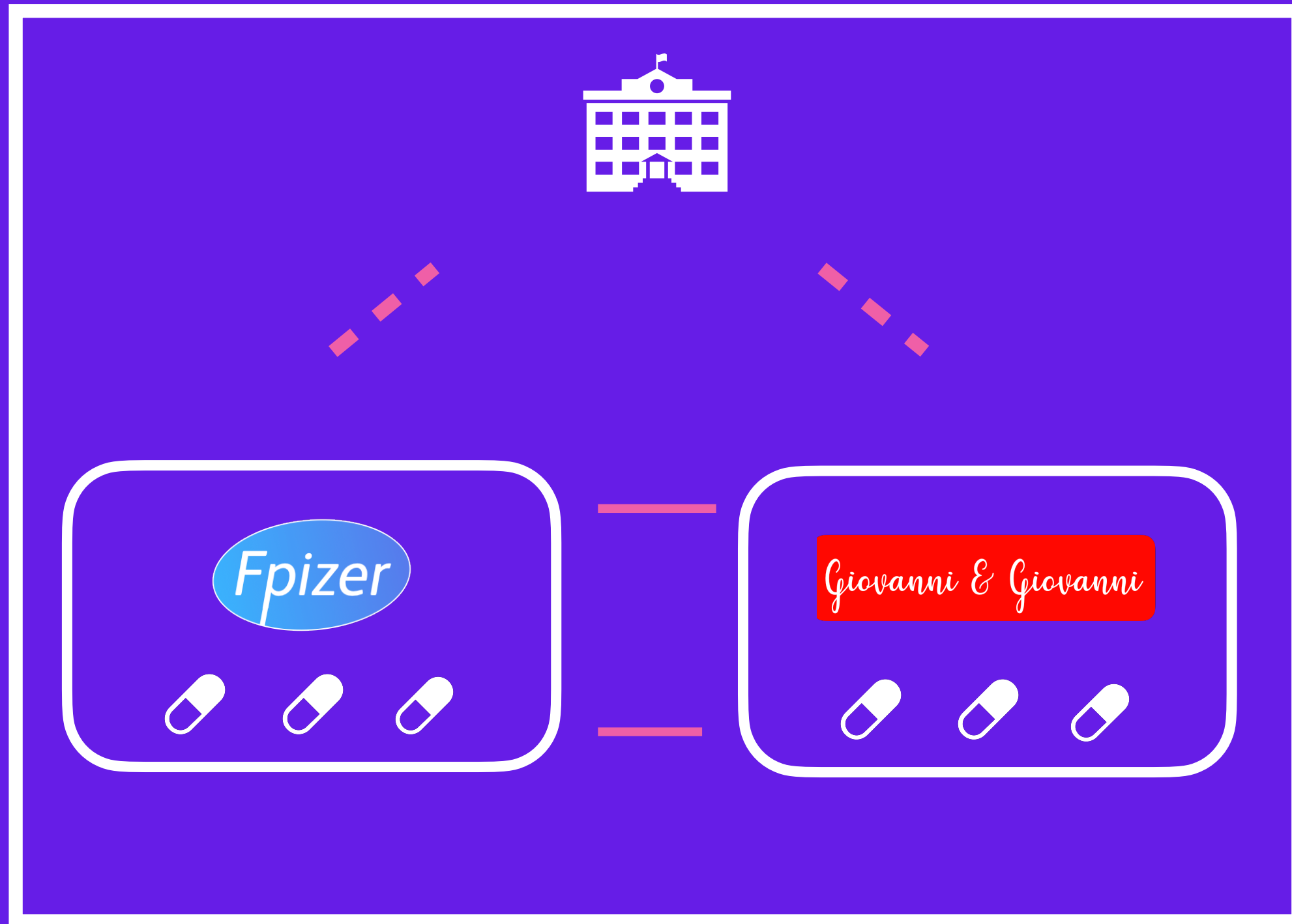
Fpizer



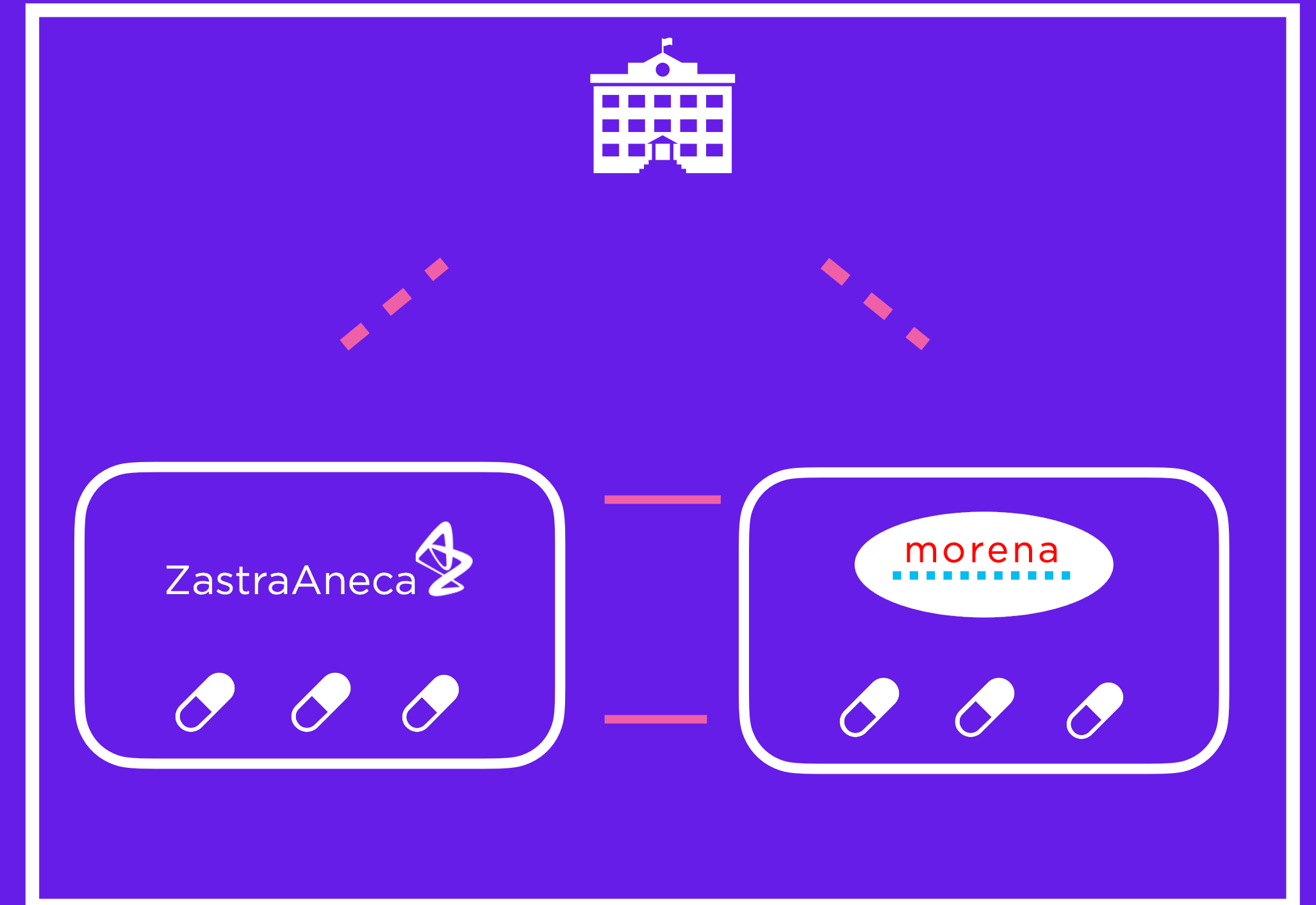
Giovanni & Giovanni



Canada



The US



Canada interacts with the US

The countries exchange the drugs via imports/exports

What if....



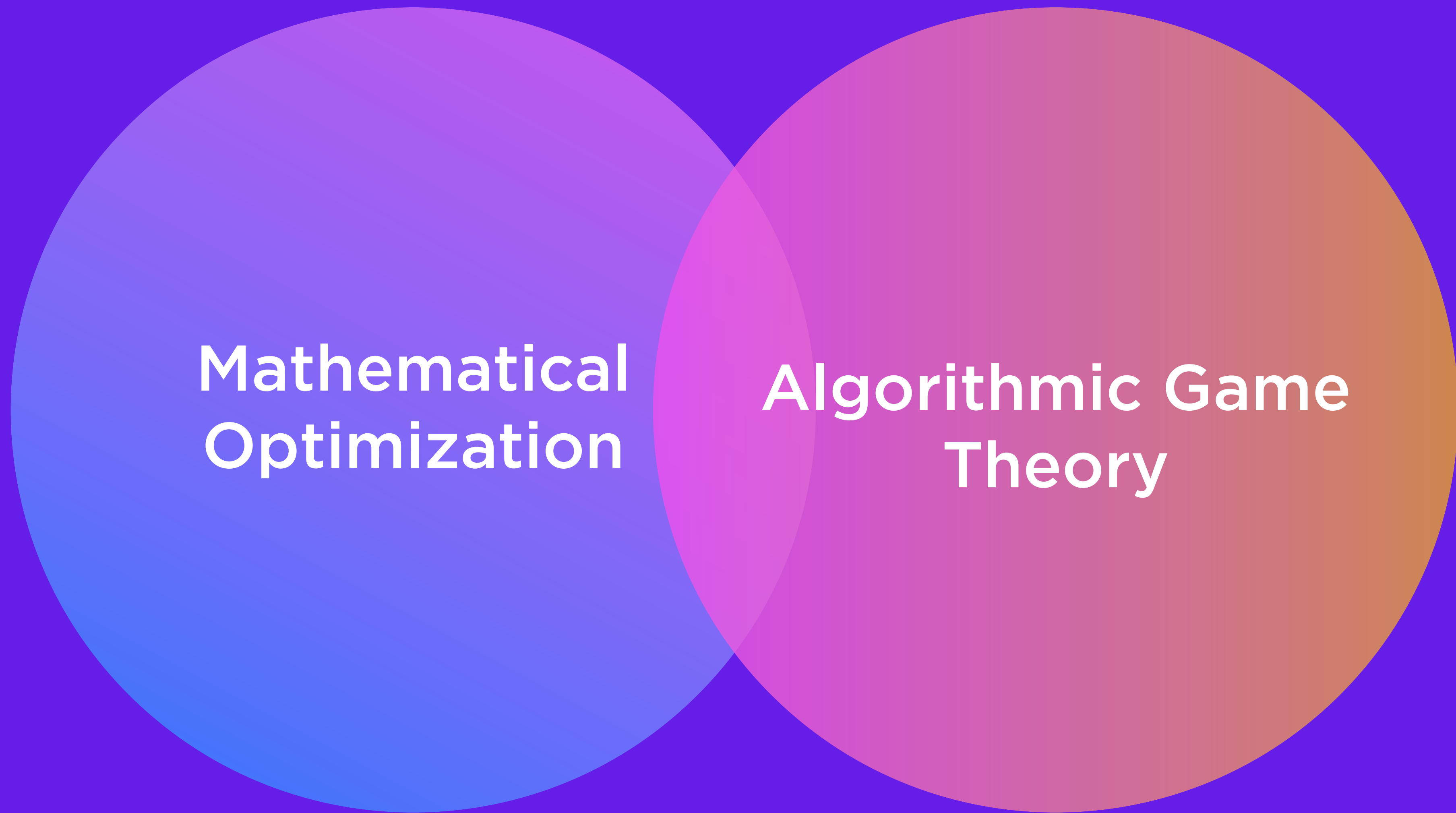
Drug companies are instead *energy producers*?



Decision-making is *rarely* an individual task.

Uncertainty

Interactions with other decision-makers



**Mathematical
Optimization**

**Algorithmic Game
Theory**

Decision-making

Strategic Decision-making

Mathematical Optimization (Programming)

The “Science of Better”

Mathematical optimization models **complex** situations and provide an efficient **decision-making tool**

Algorithms, Mathematical Theory, and Applications

However, it often ignores what “other” decision-makers are doing...

The “Science of Interaction”

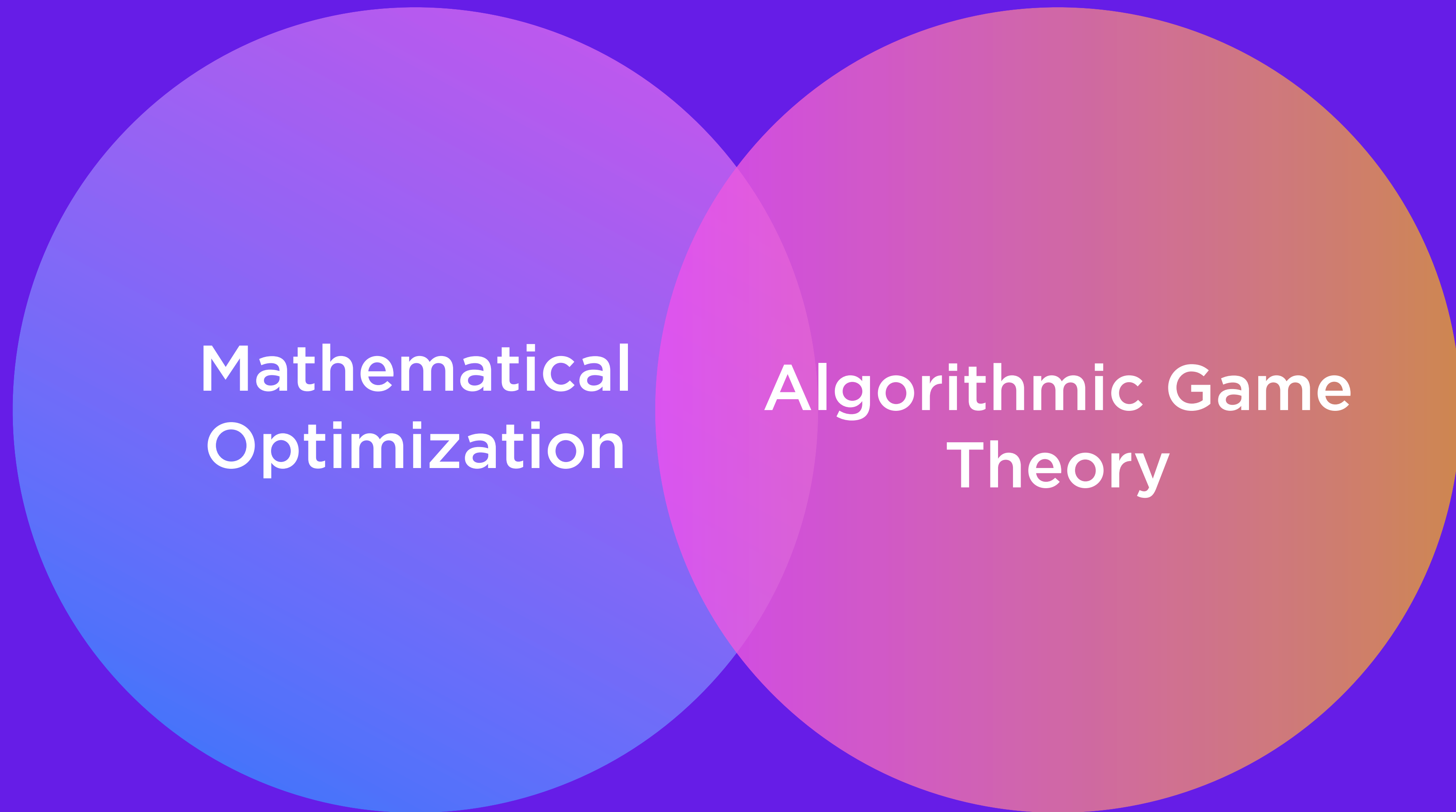
Algorithmic Game Theory is the study of **strategic behavior** among competitive agents (e.g., algorithms or people)

However, it often **ignores the underlying complexity** behind decisions



Algorithmic Game
Theory

Mathematical Programming Games



“Complex” Strategic Decision-making

Mathematical Programming Games

I study **mathematical models**

among agents that solve **complex optimization problems**

Commute from
Princeton to NYC



“Shortest Path”
problem

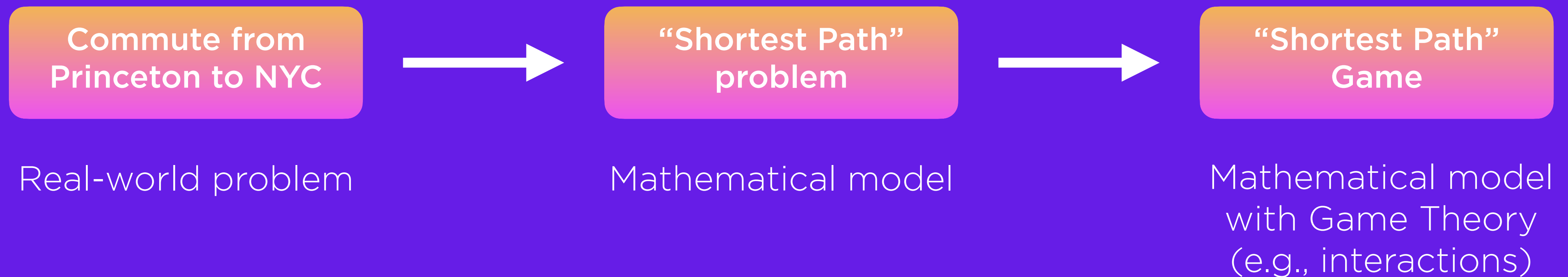
Real-world problem

Mathematical model

Mathematical Programming Games

I study **mathematical models**

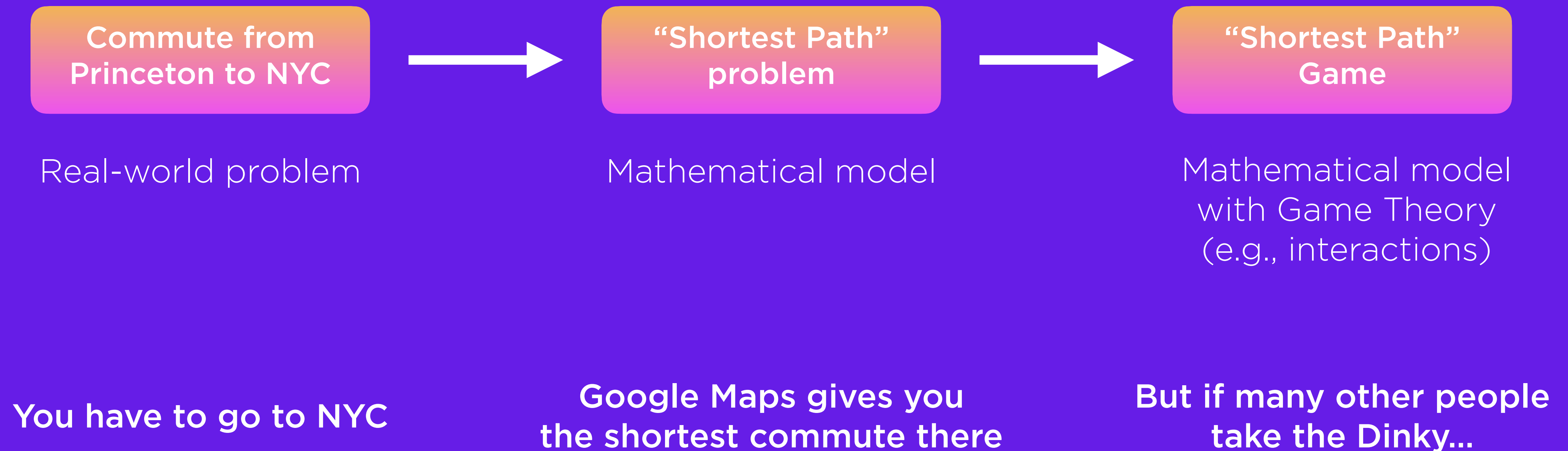
among agents that solve **complex interconnected optimization problems**



Mathematical Programming Games

I study **mathematical models**

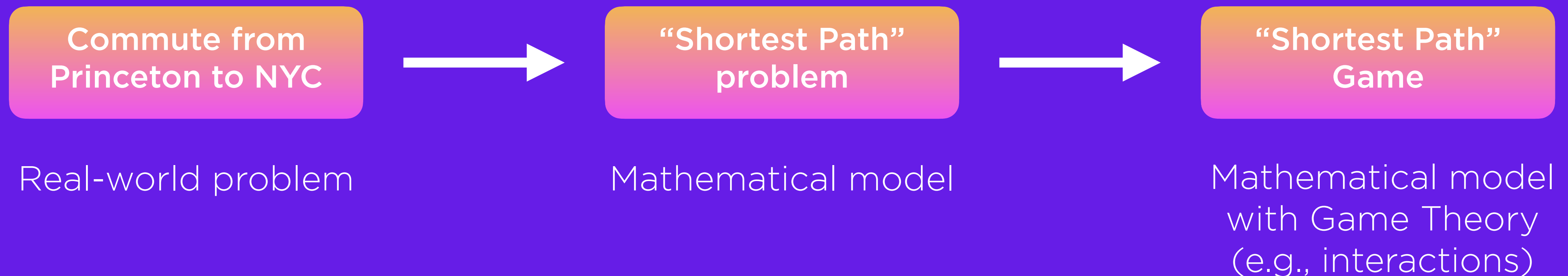
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Mathematical Programming Games

I study **mathematical models**

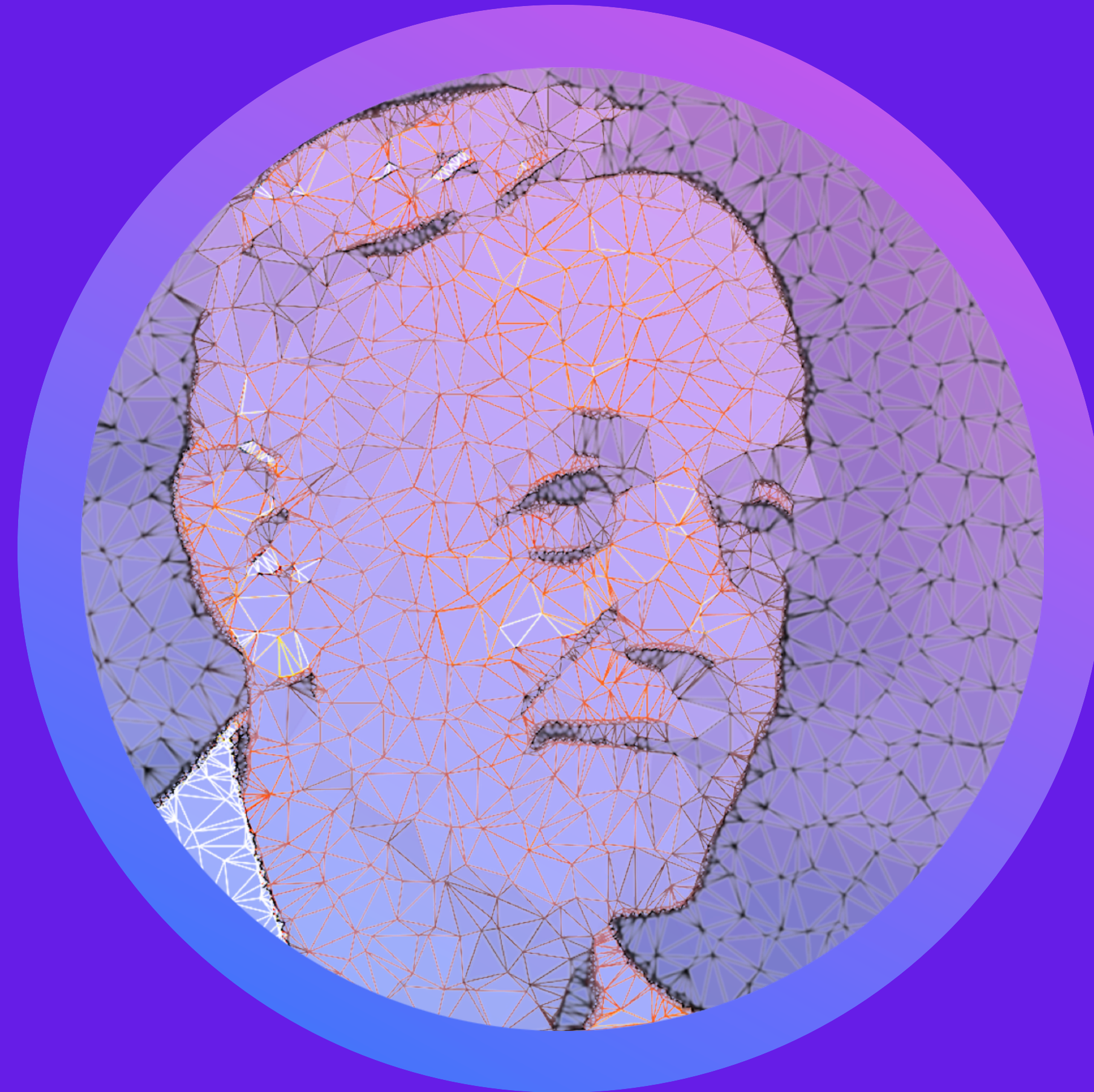
among agents that solve **complex interconnected optimization problems**

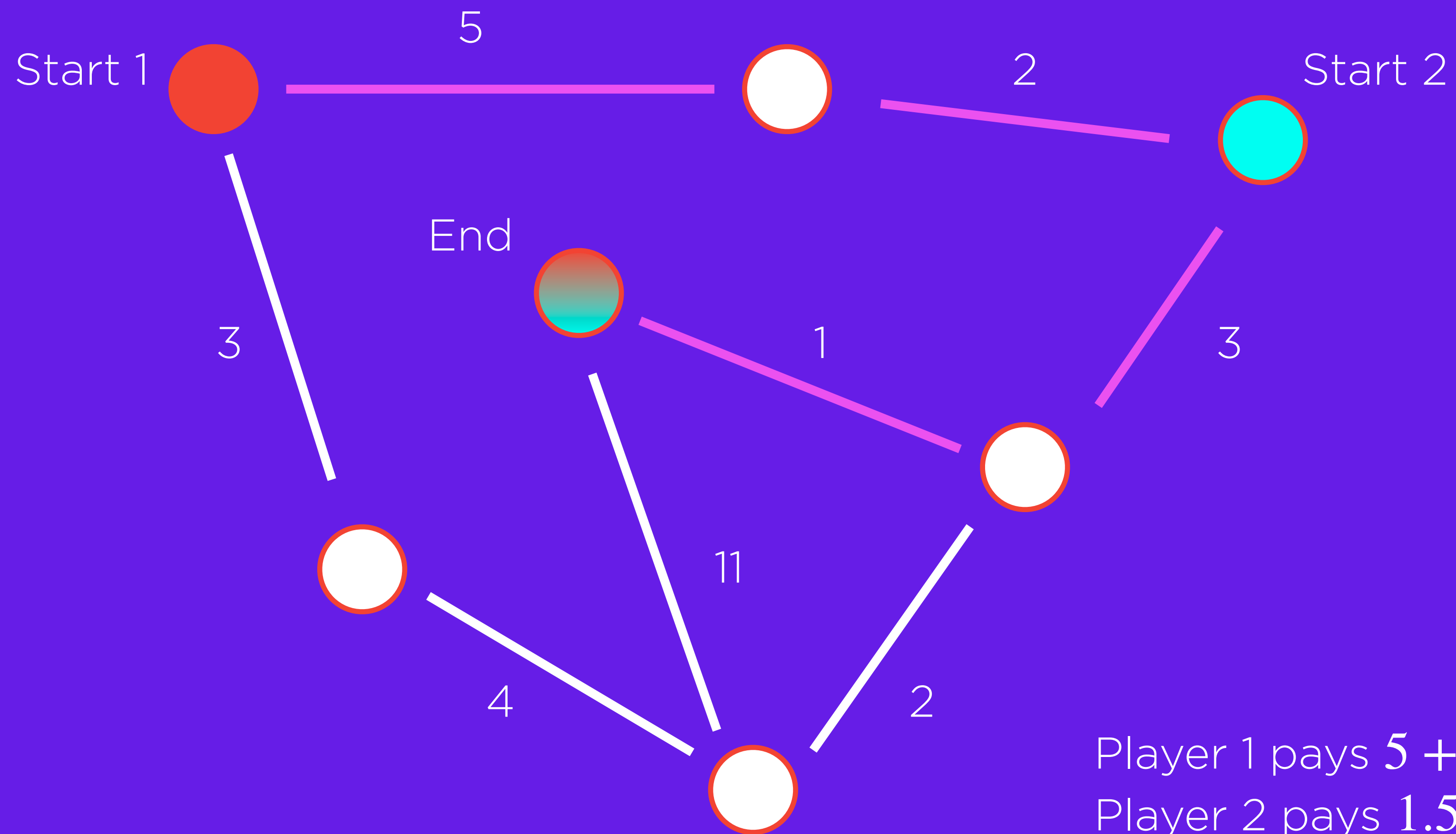


I build algorithms to **balance individual selfishness** with the **goals of the collectivity**

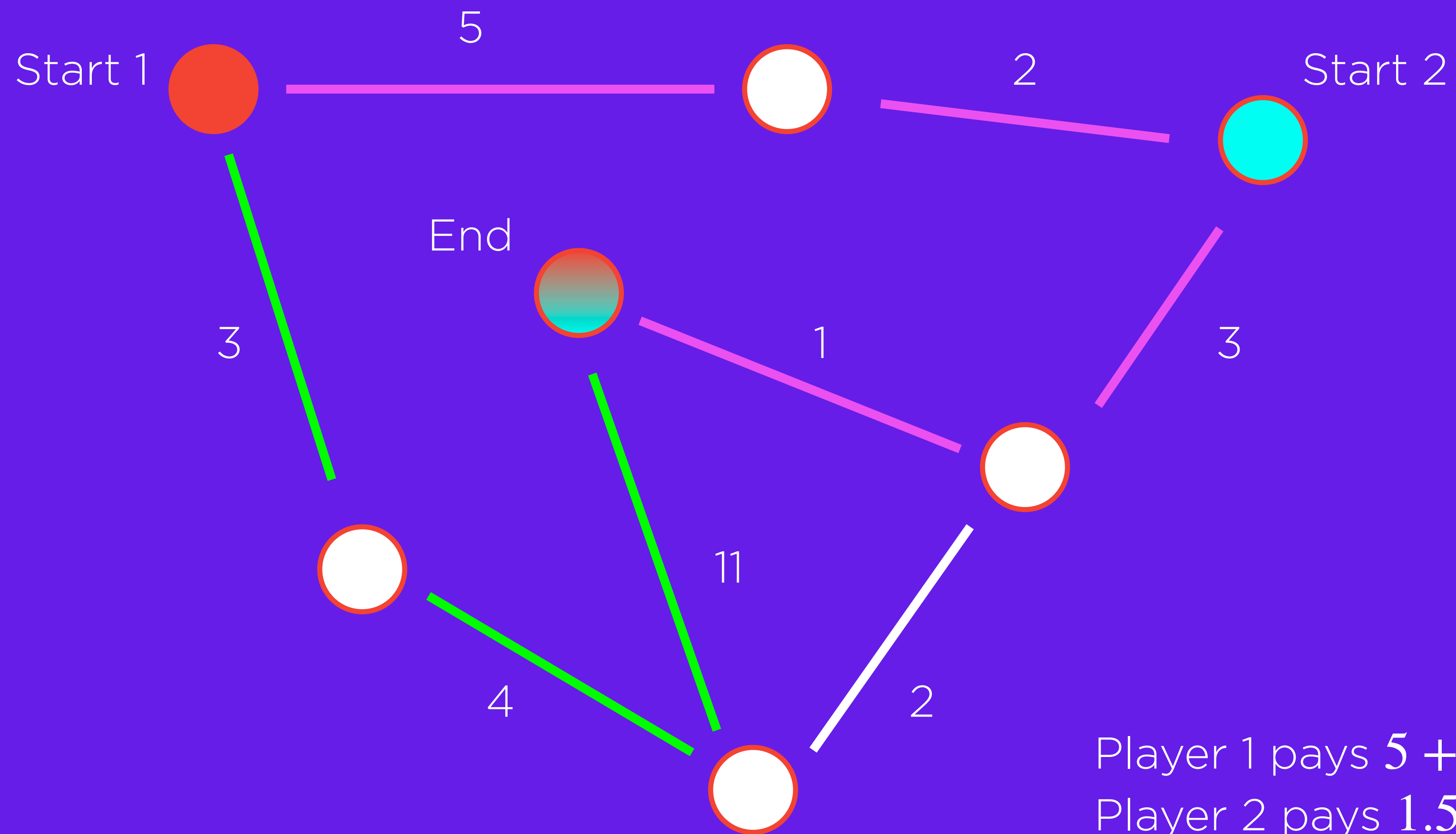
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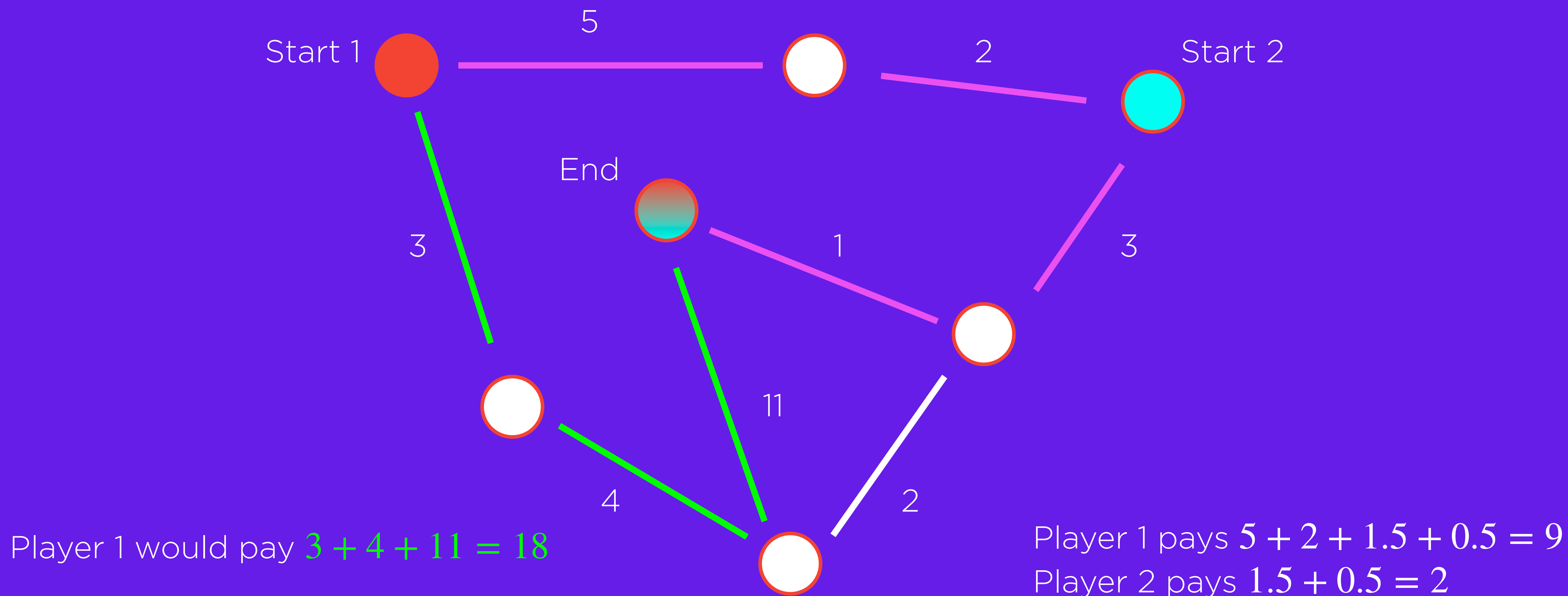
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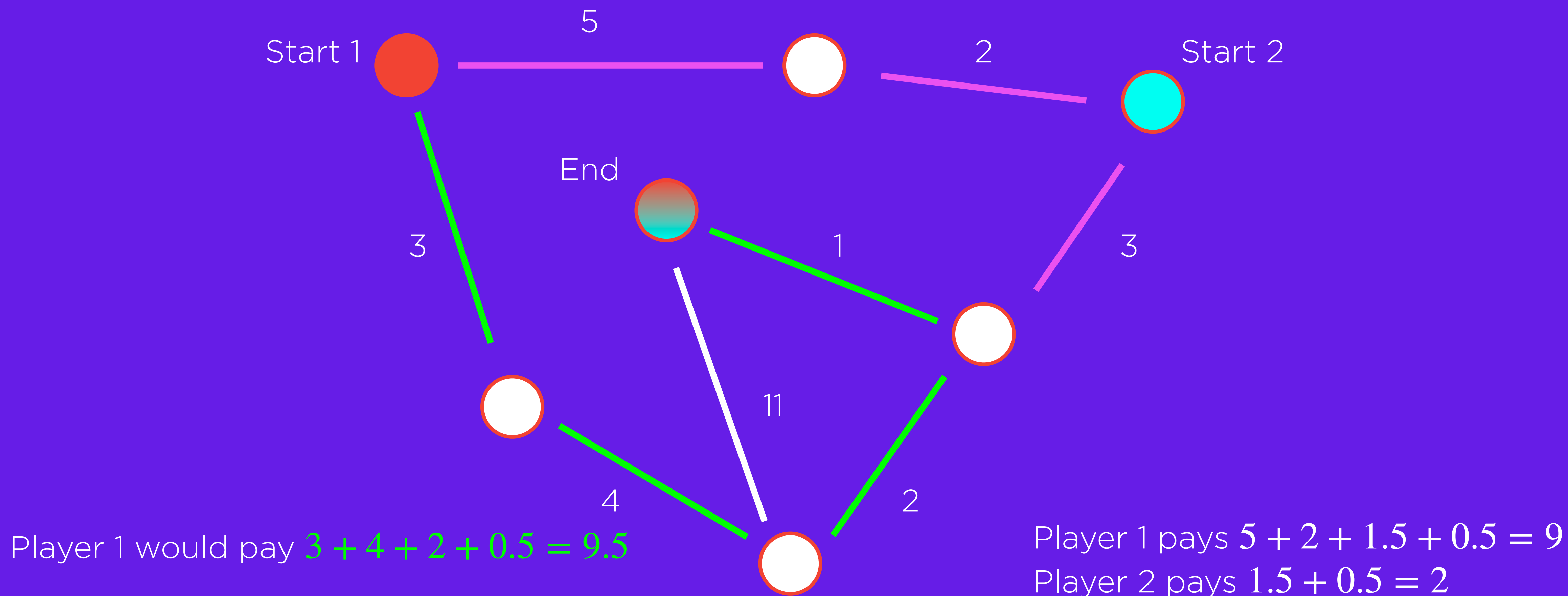




Player 1 pays $5 + 2 + 1.5 + 0.5 = 9$
Player 2 pays $1.5 + 0.5 = 2$







Why is this important?

Algorithms are increasingly permeating our world

A photograph of a wind farm with several large wind turbines. The image is overlaid with a semi-transparent purple filter. The text "Algorithms are increasingly permeating our world" is centered in white.

Algorithms are increasingly permeating our world

Energy systems



Algorithms are increasingly permeating our world

Energy systems

Commuting

Algorithms are increasingly permeating our world

READY FOR SERVICE

1993

NUMBER OF
CABLES LAID

4

Energy systems

Commuting

Internet Infrastructure



Algorithms are increasingly permeating our world

Energy systems

Commuting

Internet Infrastructure

Retail decisions



Algorithms are increasingly permeating our world

Energy systems

Commuting

Internet Infrastructure

Retail decisions

Health decisions

Algorithms are increasingly permeating our world

Algorithms are increasingly permeating our world

The future of our autonomous decision-making critically depends upon our ability to **understand and incentivize algorithms to cooperate** despite their **selfish and diverse objectives**

Algorithms and Mathematics that can
improve other algorithms



Thank you!

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