

When Nash meets Stackelberg: Nash Games Among Stackelberg Leaders

*M. Carvalho, G. Dragotto,
F. Feijoo, A. Lodi, S. Sankaranarayanan*

CANADA
EXCELLENCE
RESEARCH
CHAIR



DATA SCIENCE
FOR REAL-TIME
DECISION-MAKING

POLYTECHNIQUE
MONTREAL
TECHNOLOGICAL
UNIVERSITY





Energy Applications

Connections to Pascal's and Dorothee's talks



Consider a ***Ski*** shop



Merlin

sells *skis* in a **market** in order to **profit**



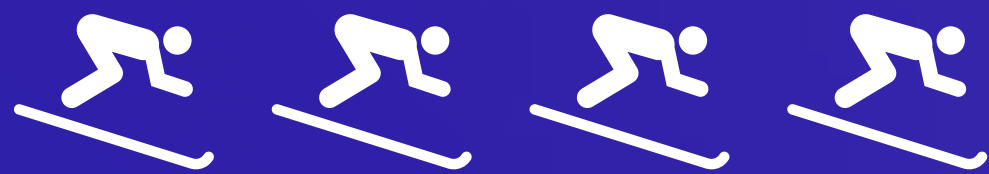
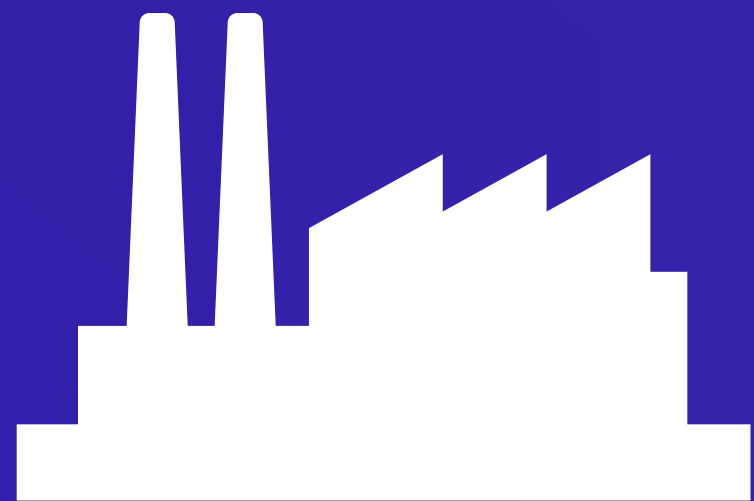
Nash Game



And competes with **Leon Sports**
Hence, Leon Sports and Merlin are **playing a Nash Game**



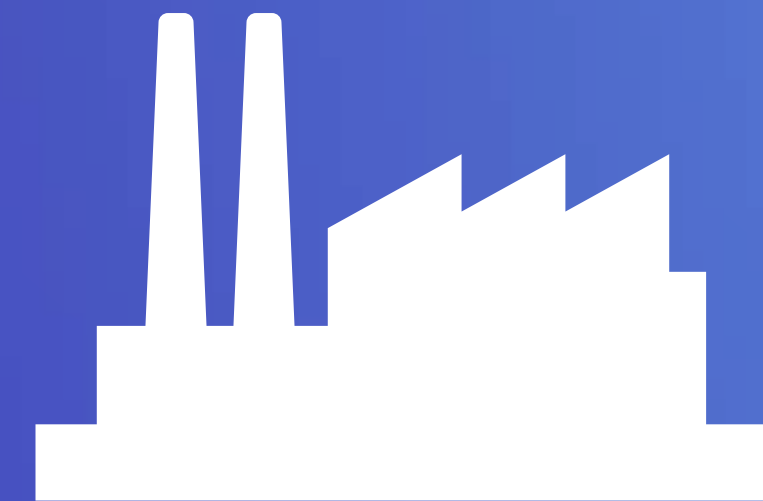
Aussois Ski Resort
taxes and regulates the *ski market*



Merlin



Nash Game



Leon Ski



Aussois Ski Resort

Stackelberg Games




Ski Corporation A

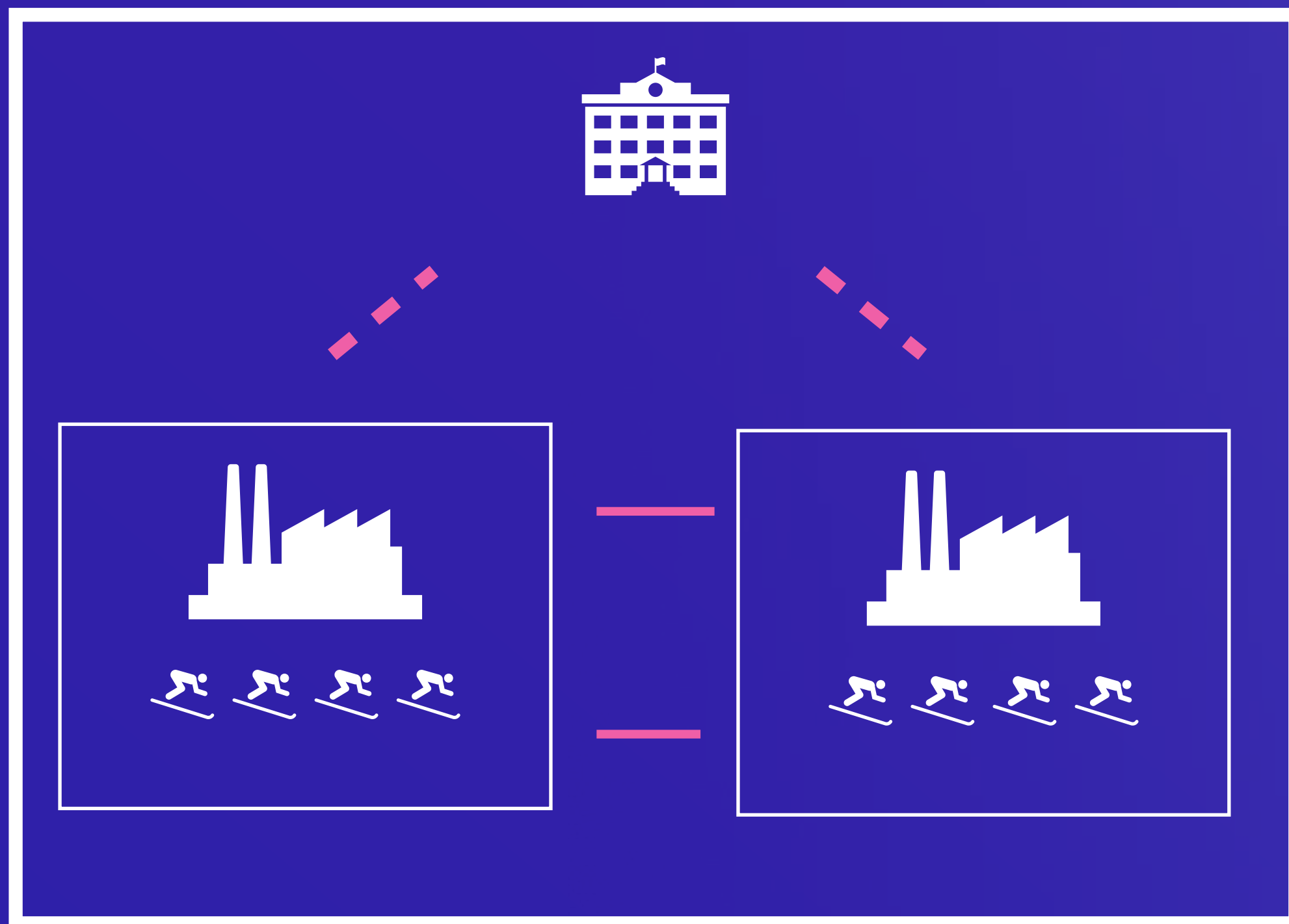
Nash Game




Ski Corporation B

Aussois Ski Resort

Plays a **Stackelberg** game with Ski-Rentals



Nash Game



Aussois Ski Resort trades skis and competes with **Bardonecchia**

We call this a Nash Game Among Stackelberg Leaders (***NASP***)

Where do we stand

Theorem (Carvalho, D., Feijoo, Lodi, Sankaranarayanan, 2019)

Given a (trivial) NASP with **2 leaders** and **1 follower** each, where the **followers** solve a **linear program** and the **leaders** have **linear objectives**:

1. It is Σ_2^P -hard to decide if the problem has a **Mixed Nash Equilibrium**, and
2. It is Σ_2^P -hard to decide if the problem has a **Pure-Strategy Nash Equilibrium** even if all feasible regions are **bounded**.

 **There is still hope!**

NASPs have a well-defined **polyhedral structure**

Even though **non-convex**, we can use Balas' *clconv* to "**convexify**"

We have a  **full of algorithmic tools** for finding equilibria with guarantees/certificates

Convexification in a Game Theoretic context

*MNE over the **convex-hull** = MNE for the **original** game!*

Let's talk about this 😊

